Teaching the Postulates of Quantum Mechanics in High School: A Conceptual Approach Based on the Use of a Virtual Mach-Zehnder Interferometer

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Abstract
In this article, we propose a conceptual approach to discuss the postulates of quantum mechanics in high school level. The idea is to provide a ‘translation’ from quantum formalism to an accessible language for high school students in which the postulates are presented on a conceptual-phenomenological basis. Instead of using mathematical formalism, we shall illustrate some of the quantum postulates by focusing on a virtual simulation of the Mach-Zehnder interferometer.

Introduction
For decades, studies on physics education have been concerned about how to teach in appropriate way quantum mechanics at high school. Most of this works are motivated by both conceptual and mathematical difficulties associated with quantum theory. As Hoekzema et al (2007) asserted, with many of conceptual difficulties being unavoidable, simplifying the mathematics becomes a top priority. Some authors have focused on the uncertainty principle (e.g. Johansson and Milstead 2008). Others have emphasized the de Broglie’s equation for deducing the energy level of a particle in a box, a finite square well, the hydrogen atom and a harmonic oscillator (e.g. Gianino 2008).

As a complement to these works, we present an instructional approach based on the canonical formulation of quantum theory in which six postulates play a central role. We shall propose a ‘translation’ from quantum formalism to an accessible language for high school students in which the postulates are presented on a conceptual-phenomenological basis. Instead of making statements about kets, bras, operators and others abstract mathematical entities, we shall describe the quantum postulates in terms of concepts associated with physical reality such as state, eigenstates, eigenvalues and observables. In addition, the notions of superposition, collapse, probability and time evolution are also introduced. Thus, mathematical formalism is avoided by using simulation software assistance.

The software here involved is a virtual simulation of the Mach-Zehnder interferometer, developed by our research group (Pereira et al 2009). The Virtual Mach-Zehnder Interferometer (VMZI) illustrates the interference of photons by simulating a light beam consisting of singles photons. Real experiments with singles photons have been performed since the beginning of the 1980s in advanced researches in Physics. Some didactical versions of these experiments have been developed for undergraduate level (Galvez et al 2005). Unfortunately, the proper technological resources required for these experiments are too much expensive for most schools, which makes almost impossible to demonstrate quantum interference in high school level. We believe that VMZI can fill this gap.
The Mach-Zehnder interferometer

The Mach-Zehnder interferometer is a simple optical device created independently by Ludwig Zehnder (1854-1949) and by Ludwig Mach (1868-1951) around 1891-1892. It demonstrates the light interference by division of amplitudes (Zetie et al. 2000). In the figure 1, a light beam is split into two components, A and B, by a beam splitter BS1. Each one of these components is reflected by a mirror, M1 and M2. A second beam splitter BS2 subdivides each of these components into two subcomponents and then recombines them before hitting the detectors D1 and D2. For an incident angle of 45°, each reflection causes in the light wave a phase shift of $\frac{\pi}{2}$, which corresponds to a path length difference of $\frac{\lambda}{4}$, where $\lambda$ is the wavelength of the light beam (Degiorgio 1980). In D1 direction, both subcomponents involved are reflected twice, which makes them to remain in phase, interfering constructively. In D2 direction, by the other hand, one of these subcomponents (path B) is reflected three times while the other one (path A) is reflected only once. The phase difference between them turns to $\pi$, which corresponds to a path difference of $\frac{\lambda}{2}$ (destructive interference). As result of this experiment, a light beam is projected in D1 and nothing is detected in D2.

![Figure 1. Scheme of Mach-Zehnder Interferometer.](image)

According to some authors (Adams 1998, Pessoa Jr. 2003, Scarani 2006), the Mach-Zehnder interferometer can be a powerful tool for discussing fundamental concepts of quantum mechanics. In that case, we just have to consider a beam of light consisting of only a single photon. This consideration naturally leads to the path-choice problem (Scarani and Suarez 1998). In can help students to discover from the very beginning how quantum phenomena deviate from our classical everyday experience (Müller and Wiesner 2002). Although it has being largely used in nonlinear optical researches and technological applications (for example, see Kanseri et al. 2008), the Mach-Zehnder interferometer is rarely mentioned in Physics textbooks, which makes it quite unfamiliar for most high school physics teachers.

The postulates of quantum theory

In quantum mechanics the ‘state’ represents a set of complete information about the physical system. The ‘observable’ (eg. momentum or energy), represents the measurable physical quantities of the system. All possible results of a measurement are defined as ‘eigenvalues’ of the observable being measured. The physical states associated with these eigenvalues are the ‘eigenstates’. They correspond to mutually exclusive alternatives.
Incompatible observables such as position and momentum do not have a complete set of simultaneous eigenstate. This statement is known as the uncertainty principle.

According to Cohen-Tannoudji (1977), the postulates of quantum mechanics can be stated as the following:

**First Postulate**: At a fixed time $t_0$, the state of a physical system is defined by specifying a ket $|\psi(t_0)\rangle$ belonging to the state space $\mathcal{E}$.

Since $\mathcal{E}$ is a vector space, the first postulate implies a superposition principle: a linear combination of state vectors is a state vector.

**Second Postulate**: Every measurable physical quantity $A$ is described by an operator $\hat{A}$ acting in $\mathcal{E}$; this operator is an observable.

**Third Postulate**: The only possible result of the measurement of a physical quantity $A$ is one of the eigenvalues of the corresponding observable $\hat{A}$.

**Fourth Postulate**: When the physical quantity $A$ is measured on a system in the normalized state $|\psi\rangle$, the probability $P(a_n)$ of obtaining the non-degenerate eigenvalue $a_n$ of the corresponding observable $\hat{A}$ is:

$$P(a_n) = |\langle u_n | \psi \rangle|^2$$

where $| u_n \rangle$ is the normalized eigenvector of $\hat{A}$ associated with the eigenvalue $a_n$.

**Fifth Postulate**: If the measurement of the physical quantity $A$ on the system in the state $|\psi\rangle$ gives the result $a_n$, the state of the system immediately after the measurement is the normalized projection of $|\psi\rangle$ onto the eigensubspace associated with $a_n$.

**Sixth Postulate**: The time evolution of the state vector $|\psi(t)\rangle$ is governed by the Schrödinger equation:

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = H(t) |\psi(t)\rangle$$

where $H(t)$ is the observable associated with the total energy of the system.

Many of these postulates establish a link between physical reality and mathematical formalism, as it is shown in figure 2.

![Figure 2. Theoretical ‘link’ between physical reality and mathematical formalism.](image-url)
In order to avoid mathematical formalism, we propose the replacement of these quantum postulates by six rules concerning only the domain of physical reality. These rules are based on a conceptual description of the measurement process as asserted by Dirac (1947) and by Sakurai (1994). The conceptual-phenomenological version of the quantum postulates is formulated as the following.

**Rule 1**: Before a measurement, a system is assumed to be in a superposition of state (a linear combination of its eigenstates).

**Rule 2**: A measurement usually changes the state of the observable being measured, unless the state is already in one of its eigenstates.

**Rule 3**: The result of a measurement yields one of the eigenstate of the observable being measured. The measurement process selects one of its eigenstate and rejects all the others.

**Rule 4**: The laws of quantum mechanics do not predict the result of a measurement but only the probability of the system for jumping in one of its eigenstate.

**Rule 5**: Repeated measurements of the same observable in succession yield the same result.

**Rule 6**: The state of a physical system changes with time.

**An illustration with VMZI**

Many important statements of these six rules can be conceptually shown on the VMZI. Figure 3 shows the layout of the VMZI operating in a simple configuration. Single photons are shot into the interferometer, one at time. Two detectors, the green one and the red one, are placed in each path of the interferometer in order to perform a measurement. In this case, the photons’ behavior must be described in terms of its *translational state* (Dirac 1947).

Figure 3. Photon detection in the green detector (a red light flashes on top of this detector).
In this case, the photon is described as going partly into each of the two components into which the incident beam is split. In other words, the photon can be thought as a combination of reflected and transmitted eigenstates, which means that the translational state of the photon is given by the superposition of the two translational states associated with the two components. By determining the energy in one of the two components, we obtain either the whole photon or nothing at all. It means that after hitting the detectors, the detection forces the photon to jump into one of its eigenstates (path 1 or path 2). Thus, the photon changes suddenly from being partly in one component and partly in the other to being entirely in one of the two components. This collapse is the result of the disturbance in the translational state of the photon caused by the measurement process. Before the measurement we are unable to determine which path the photon will take. We only can determine the probability of finding the photon in one particular path.

Figure 4. Interference of photons

Conclusion

In this article, an instructional approach for the teaching of quantum mechanics in high school level is proposed. This approach is based on the canonical formulation of quantum mechanics in which six postulates play a central role. We formulated a conceptual version of these postulates in which mathematical formalism is avoided. Some of these postulates were discussed by using a Virtual Mach-Zehnder Interferometer (VMZI).

As an illustration of simulation software assistance, we presented only one simple experiments with the VMZI in which a detection of a single photon are described in term of the postulates of quantum theory. Nevertheless, the use of VMZI proposed here also involves discussions on polarization and interference of photons (see figure 4 and figure 5). This instructional approach is being tested in a secondary school with a group of high school students. The results of this didactical intervention may be available by November 2010.

A new version of VMZI is being developed by our research group. The idea is to include nondemolition detector to discussing issues such as the fifth postulate and the measurement problem. In this new version, A KDP crystal may be available for replacing the first beam splitter in order to demonstrate interference of two entangled photons. The version of VMZI used here is available at the following Web address: www.if.ufrgs.br/~fernanda
Figure 5. Polarization of the photons.

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