ABSTRACTS

Евгений Фейгин - Evgeny Feigin, *Semi-infinite Pluecker relations*

Classical Pluecker relations describe the projective embeddings of the flag varieties of the group *SL(n)*. They have very rich representation-theoretic, geometric and combinatorial structure. I will talk about the semi-infinite analogue of the Pluecker relations with the basic field replaced by the Taylor series in one variable. The resulting objects (algebraic, combinatorial and geometric) are infinite-dimensional and play prominent role in several fields of modern mathematics.

Василий Горбунов - Vassili Gorbounov, *Electrical mathematics*

The theory of electrical networks was laid out by Kirchhoff in 1847 in a short paper where he calculated the conductance of a linear resistive electrical network. His ideas were profound. In the lecture we will outline the amazing connections of his work to the disciplines like discrete harmonic analysis, enumeration on graphs, representation theory of quantum groups, theory of totally positive matrices, cluster algebras, integrable systems and even neural networks in as elementary form as it is possible. We will indicate the possible directions for research in this modern and active area of mathematics.

Валентина Кириченко - Valentina Kirichenko, *Newton-Okounkov convex bodies and representation theory*

Theory of Newton-Okounkov convex bodies extends methods of toric geometry to non-toric varieties. In the case of flag varieties, several well-known families of polytopes from representation theory (such as Gelfand-Zetlin and Feigin-Fourier-Littelmann-Vinberg polytopes) can be obtained as Newton-Okounkov polytopes. I will give an introduction to the theory of Newton-Okonkov convex bodies and then focus on the following applications to representation theory:

(1) Geometric valuations on flag varieties;
(2) A uniform valuation on flag varieties for classical groups;
(3) Explicit description of Newton-Okounkov polytopes in types *A* (flag varieties *for SL(n)*) and *C* (flag varieties for *Sp(2n)*);
(4) Examples in types *B* and *D* (flag varieties for *SO(n)*) and open problems.

小林俊行 − Toshiyuki Kobayashi, *Symmetry breaking operators: general theory and concrete construction for reductive groups*

The lectures highlight infinite-dimensional representations of real reductive groups with focus on induction and restriction. I begin with the general results such as geometric criterion for finiteness (of uniform boundedness) of the multiplicities of irreducible representations occurring in induction and restriction.

Symmetry breaking operators are morphisms that describe restriction of representations. In the second lecture, I plan to discuss construction problem of "symmetry breaking operators" in a concrete geometric setting.

Sophie Morier-Genoud, *Continued fractions and combinatorics*

Continued fractions form a classical area in mathematics and a great variety of approaches and applications can be found in the literature. For this lecture(s) we will develop a simple combinatorial approach. The main ingredients are triangulations of polygons and more generalized polygon dissections. With this approach one can cover several results which are intrinsically related to each other, but appear separately in the literature. Among the theorems we will discuss are that of Hirzebruch (relating « regular » and « negative » continued fractions), Coxeter and Conway-Coxeter (relating continued fractions and projective geometry), and that of Series (relating continued fractions to the hyperbolic plane). More general polygon dissections appear when extending these theorems for elements of the modular group PSL(2,Z).

Валентин Овсиенко - Valentin Ovsienko, *q-deformed rational numbers*

What is a « quantum » number? This naive question has a very clear answer in the case of integers. Quantum, or *q*-integers are related to combinatorics and mathematical physics. One classical subject leading to *q*-integers is called « *q*-series » (Euler, Jacobi, Dyson,?). Another subject based on *q*-integers is the theory of Gaussian q-binomial coefficients. Many different mathematical notions have q-analogs, but, surprisingly, the notion of « quantum rationals » is completely unexplored. I will explain one possible version of *q*-rationals, which is based on a combinatorial approach to continued fractions. The theory is at its embryonic phase of development and offers much more open questions than answers. Some of these questions will be discussed.

Angela Pasquale, *Symmetric spaces, Fourier analysis and resonances*

The aim of these lectures is to introduce the notion of resonances for the Laplacian of a Riemannian symmetric space *X* of the noncompact type.
The adjective ``symmetric'' indicates that there is a Lie group acting transitively on *X*. As a first step, we introduce the Riemannian symmetric spaces of the noncompact type and their structural properties.
Our main examples are the hyperbolic spaces. The next step is to introduce an appropriate Fourier transform for (sufficiently nice) functions on *X*. This is the so-called Helgason-Fourier transform. We focus on the L^2-theory. The main results are the Paley-Wiener and the Plancherel theorems.
The last ingredient we need is the Laplacian Δ acting on the space $L^2(X)$ of L^2 functions on *X*. It is the most important among the differential operators which are invariant under the Lie group of symmetries of *X*. It is an essentially self-adjoint, positive-definite unbounded operator with purely continuous spectrum $\sigma(\Delta)=[\rho_X^2, \infty)$, where ρ_X^2 is a positive constant depending on the geometry of *X*. The resolvent $R(z) = (\Delta-z)^{-1}$ is a holomorphic function of the variable z in $\mathbf{C}\backslash\sigma(\Delta)$ (the complement of the spectrum in the complex plane), with values in the space of bounded linear operators on $L^2(X)$.

If we consider the resolvent R(z) not as an operator on $L^2(X)$ but on a dense subspace, like the space $C_c^\infty(X)$ of compactly supported smooth functions on *X*, then a continuation of R(z) across the spectrum $\sigma(\Delta)$ might be possible. The continuation might be meromorphic rather than holomorphic. In this case, the poles of the extended resolvent are the resonances of the Laplacian operator Δ . The study of the resonances of the Laplacian on Riemannian symmetric and locally symmetric spaces (or on more general complete Riemannian manifolds with bounded geometry at infinity) has several applications in mathematical physics, spectral theory, study of dynamical systems, number theory. The basic problems are to determine the existence of resonances, their location and to interpret them. For Riemannian symmetric spaces of the noncompact type, the Plancherel and Paley-Wiener theorems allow us to write an explicit integral formula for the resolvent. Understanding the resonances of the Laplacian means understanding the meromorphic continuation of this singular integral. In its generality, this is still an open problem.
In some examples the extensions turn out to be holomorphic and there are no resonances. In other examples, the resonances exist and can be explicitly determined. They are then linked to the spherical principal series representations of the group of isometries of the space. In these lectures we will present the complete results for the Riemannian symmetric spaces of rank-one, like the hyperboloids, and give a short overview of the general situation.

Tomasz Przibinda, *An elementary construction of the Weil representation*

The Weil representation is a magnificent structure which keeps appearing in a variety of places throughout Mathematics and Physics. This is evident from a simple google or Mathscinet search for “oscillator representation”, “Weil representation”, “Howe correspondence” or “local theta correspondence”. The last two terms refer to a correspondence of irreducible representations for certain pairs of groups, conjectured to exist in [How79] proven to exist over the reals in [How89], over *p*-adic fields (*p* odd) in [Wal79] and essentially proven not to exist over finite fields in [AMR96]. A concise description of the Weil representation may be found in [Tho09]. Anyone interested in a short and complete presentation should read that paper and stop right there. In this lectures we take the opposite approach. We dissect the Weil representation into small pieces, study how they work, and put them back together. Hence the title of this lectures. The methods we use are elementary, *i.e.* contained in a graduate curriculum of an average university in the USA. Time permitting, we shall define Howe's correspondence, review recent results and discuss open problems.

References

[AMR96] A.-M. Aubert, J. Michel, and R. Rouquier. Correspondance de Howe pour les groupes réductifs sur les corps finis. Duke Math. J., 83, pp. 353-397, 1996.
[How79] R. Howe. θ-series and invariant theory. In Automorphic forms, representations and *L*-functions (Proc. Sympos. Pure Math., Oregon State Univ., Corvallis, Ore., 1977), Part 1, Proc. Sympos. Pure Math., XXXIII, pp. 275-285. Amer. Math. Soc., Providence, R.I., 1979.
[How89] R. Howe. Transcending Classical Invariant Theory. J. Amer. Math. Soc. 2, 2:535--552, 1989.
[Tho09] T. Thomas. Weil representation, Weyl transform, and transfer factor. Preprint on author's webpage, 2009.
[Wal89] J. L. Waldspurger. Démonstration d'une conjecture de dualité de Howe dans le cas *p*-adique, p≠2. Festschrift in honor of I. I. Piatetski-Shapiro on the occasion of his sixtieth birthday, Part I, Israel Math. Conf. Proc., 2, pages 267-324, 1989.

Евгений Смирнов - Evgeny Smirnov, *Schubert polynomials and toric degenerations of flag varieties*

Schubert polynomials were defined about 40 years ago by Bernstein, Gelfand and Gelfand and independently by Lascoux and Schuetzenberger. This is a basis in the ring of polynomials in countably many variables with integer coefficients, indexed by finitary permutations. These polynomials represent the cohomology classes of Schubert varieties in the cohomology ring of a full flag variety and enjoy a number of nice combinatorial properties. This makes them an interesting object of study both for geometers and combinatorialists.
My first lecture will be mostly devoted to combinatorics of Schubert polynomials; we will discuss their definitions, positivity of their coefficients, relation to Schur functions etc. In my second lecture, I will explain their relation to toric degenerations of flag varieties, due to Knutson and Miller. Time permitting, I will also mention a partially conjectural generalization of this story to another basis in the ring of polynomials: slide polynomials, defined by Assaf and Searles, and some open problems related to this.

Владлен Тиморин - Vladlen Timorin, *Combinatorics of Gelfand-Zetlin polytope*

There are several remarkable series of convex polytopes arising in representation theory. Combinatorial properties of these polytopes are important to understand for two reasons: 1) we need invariants to distinguish different polytopes or to have evidence that different descriptions give rise to the same polytope, 2) there is a deep analogy between combinatorics and algebraic geometry; a representation theory setting often provides both a polytope and an algebraic variety; the combinatorial quantities associated with the polytope often translate into enumerative geometry quantities associated with the variety. Gelfand-Zetlin polytopes are associated with representations of the special linear groups/algebras and with varieties of complete flags.
I will overview some results concerning combinatorics of Gelfand-Zetlin polytopes, in particular:

1) Partial computation of f_k= the number of k-faces
2) Ladder diagrams
3) Diameters and automorphisms.

柳田伸太郎 − Shintaro Yanagida, *Introduction to Hall Algebras*

I will give an introductory talk on Hall algebras. In the first part, I will explain the definition and basic properties of Ringel-Hall algebra from scratch, and give a few examples including the classical Hall algebra, Hall algebras for Dynkin quivers and Hall algebras for smooth projective curves. I will decide the content of the second part according to the preference of the audience. Candidates are

1) Toën's derived Hall algebra and its relation to vertex algebras
2) Bridgeland's Hall algebra and the Drinfeld double
3) Lusztig's geometric Hall algebra.