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Bayesianism and the Common Prior Assumption in Game Theory

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Abstract:

The epistemic program in game theory consists in introducing Bayesian decision theory into a game-theoretic framework. Robert Aumann's claim that the solution concept of correlated equilibrium is the expression of Bayesian rationality is one the most significant results of the epistemic program for social and behavioral sciences. This claim depends on the controversial common prior assumption. This paper evaluates the philosophical and theoretical status of this assumption with regards to Bayesianism, *i.e.* the doctrine that Bayesian decision theory is always rational. I argue that Aumann's defense of this assumption is purely formal and that it is actually hard to give it any meaningful substantive interpretation. It follows that the implications of Bayesianism in game theory are rather unclear. I provide an alternative account of the common prior assumption in terms of symmetric reasoning that may open the way for a non-Bayesian version of the epistemic program in game theory.

Keywords : Bayesianism; Common prior assumption; Correlated equilibrium; Epistemic game theory; Symmetric reasoning

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1. Introduction

As a theory of rationality, Bayesianism may be defined as the doctrine that Bayesian decision theory is always rational [1, p.13]. Bayesian decision theory may itself take several forms but in this paper it will more closely correspond to Leonard Savage's [2] axiomatic account of subjective expected utility theory. Savage defines rational behavior in terms of a set of axioms and shows that, provided these axioms are satisfied, the choice of an agent can be represented as the maximization of the expectation of some utility function unique up to positive linear transformations. The so-called "epistemic program" in game theory that emerged in the 1980's can be seen as an attempt to introduce Bayesian decision theory in a game-theoretic framework [3]. Contrary to one-person decision problems where the decision-maker has to take into account only independent states of nature, strategic interactions imply that each player must form subjective beliefs over the actions and beliefs of other players. Epistemic game theory (henceforth, EGT) is thus the branch of game theory that formalizes explicitly the players' knowledge and beliefs about others and the way they reason on their basis.

The epistemic program has led to significant results regarding the epistemic requirements related to various solution concepts such as the Nash equilibrium concept (*e.g.* [4], for an overview see [5]). This paper however is concerned with another aspect of the epistemic program, namely its potential for leading to a self-contained theory of rationality in strategic interactions. Robert Aumann's [6] claim that the solution concept of correlated equilibrium is "an expression of Bayesian rationality" is essential here. This claim depends on the controversial common prior assumption (henceforth, CPA). Herbert Gintis [7] has recently suggested that Aumann's account should serve as the key foundation for a theory of social norms encompassing all the behavioral sciences (economics, sociology, psychology and biology). Though I have elsewhere endorsed and extended Gintis' suggestion ([8], [9], [10]), I shall argue that Aumann's defense of the CPA is purely formal and that it is actually hard to give it any meaningful substantive interpretation. It follows that the implications of Bayesianism in game theory are rather unclear. However, I provide an alternative account of the common prior assumption in terms of symmetric reasoning that may open the way for a non-Bayesian version of the epistemic program in game theory.

The paper is organized as follows. Section 2 briefly expands the characterization of Bayesianism in the context of strategic interactions and states informally Aumann's claim as well as Gintis' interpretation of it. Section 3 develops a formal framework in terms of an epistemic model and formally states the CPA and its implications. Section 4 surveys a set of conceptual issues related to the introduction of Bayesianism into a game-theoretic framework and argues that the CPA has no obvious substantive meaning. Therefore, I claim that Bayesianism cannot ground a theory of social interactions. Section 5 offers an alternative account on the basis of a modified epistemic model and where the CPA is reinterpreted in terms of "symmetric reasoning". Section 6 concludes. An appendix sketches proofs for several claims.

2. Bayesianism and Strategic Interactions

As indicated above, Bayesianism can be defined as the doctrine that Bayesian decision theory is always rational. More exactly, if in a given decision problem D Bayesian decision theory holds that x is the best choice, then Bayesianism implies that x is rationally mandatory (or at least permissible). Correspondingly, we will say that someone is Bayesian rational in D whenever he chooses the option x prescribed by Bayesian decision theory. These are very general statements that do not fully characterize Bayesianism. In particular, Bayesian decision theory may take several specific and not fully equivalent forms. However, when speaking of

Bayesian decision theory, economists and game theorists generally refer to Savage's [2] axiomatic account of *subjective expected utility theory* (SEUT). In this section, I characterize Savage's SEUT and Bayesianism in the context of strategic interactions. I also states informally Aumann's claim regarding the relationship between Bayesian rationality and the concept of correlated equilibrium.

A decision problem is a function D: A x S → C where A is a set of *acts* (or actions), S a set of *states of nature*[1] and C a set of *consequences*. Formally, any combination of an act $a \in$ A and of a state $s \in$ S maps onto some consequence $c \in$ C, *i.e.* D(a, s) = c. The decision maker is endowed with a preference ordering ≥ over the set C of consequences and a probability measure π(.) over the set S of states of nature. An act then maps any state onto a consequence, *i.e.* a: S → C. Alternatively, an act can be seen as a *prospect* which we define as a probability distribution of consequences $c \in$ C. Finally, call any subset E ⊆ S of states of nature an *event*. At the most general level, a Bayesian rational decision maker in the sense of Savage[2] is an agent whose choice behavior corresponds to the maximization of the expectation of a utility function u(.) with respect to the probability measure π(.):

(1) $$\max_{a\in A} \sum_s \pi(s)u(a;s)$$

However, since Savage's treatment is axiomatic, this definition is only the result of a set of primitive principles, *i.e.* axioms. To be Bayesian rational is to behave according to these axioms. Among the seven axioms stated by Savage, some are essentially technical and others are more substantive [11].

Savage's expected utility theorem establishes that if an agent's preferences over the set A satisfies theses axioms, then they can be represented by an expectational utility function unique up to any positive linear transformation.[3] It thus demonstrates that the utility function u(.) and the probability measure π(.) can be determined simultaneously while other versions of expected utility theory (such as von Neumann and Morgenstern's or De Finetti's ones) assumed one or the other as given. Clearly, the probability measure represents the agent's subjective beliefs over the events, *i.e.* over any feature of the decision problem that is *independent* of his choice. My point here is not to assess Savage's axioms and the corresponding relevance of SEUT as a theory of rational choice.[4] My aim is rather to point out the implications of introducing SEUT into a game-theoretic framework. As I said above, this introduction is constitutive of the epistemic program in game theory. Game theorists have traditionally adopted a top-down approach in the study of solution concepts: assuming that everything is known about the behavior of other players, they usually ask what is the optimal play for each player. In normal form games, this has naturally led to give a great importance to the Nash equilibrium solution concept. EGT follows a quite different, bottom-up approach: from the perspective of each player, a game is a decision problem where there is an uncertainty over the behavior of others. The relevant issue is then to determine the epistemic requirements (*i.e.* what the players have to know and believe) for a given solution concept to be implemented in a game.

Clearly, the main difference between one-person decision problems and strategic interactions is the object of uncertainty over which the agents have to form subjective beliefs. In the former case, the Bayesian decision-maker ignores the "actual" state of nature as well as the "true"

[1] For the sake of simplicity, I assume that S is finite.
[2] I omit for the rest of the paper the precision "in the sense of Savage" or "according to Savage" when speaking of Bayesian rationality since I will not deal with other variants of Bayesian decision theory.
[3] It is worth mentioning that Savage's interpretation of his theorem is essentially behaviorist. That is, the notion of preference and the corresponding ordering ≥ are not primitives but are interpreted in terms of choices. See [12].
[4] For a critical perspective at the philosophical and theoretical levels, see [13].

probability distribution of states. However, this distribution is assumed to be independent of the decision-maker's choice.[5] Things are quite different in the case of strategic interactions studied by game theory. Here, the objects of uncertainty include (among other, exogenous objects) the choices of other players. Given that the choice of each player depends on his belief over others' choices, it appears clearly that the Bayesian decision maker also has to form beliefs over others' beliefs, and beliefs over others' beliefs over everyone's beliefs, and so on. As a Bayesian decision-maker, each player then chooses the strategy that maximizes his expected utility given this infinite hierarchy of beliefs. Clearly then, there is the possibility of an *epistemic* dependence between the players' choices and thus between acts and states of nature.[6] Even ruling out such dependence, the task of the decision-maker appears to be far more complicated than in the one-person case, as the states of nature are now widely more complex objects over which a probability measure as to be defined.

Some authors have argued that the introduction of SEUT into a game-theoretic framework leads to insurmountable analytical difficulties precisely because anything can happen, or seems to (*e.g.* [15]). Aumann's article "Correlated Equilibrium as an Expression of Bayesian Rationality" [6] is an important attempt to demonstrate that such a pessimistic conclusion is unwarranted. He establishes a theorem that can be stated informally this way:

> In a strategic interaction, Bayesian rational decision-makers sharing a common prior over the probability distribution of every relevant feature of the interaction (including their choices and beliefs) will implement a correlated equilibrium in the corresponding game.

I will formally state this theorem in the next section but before consider the following two-player game as an illustration (see fig. 1):

Fig. 1

		Bob	
		C	**D**
Ann	**C**	4 ; 4	1 ; 5
	D	5 ; 1	0 ; 0

Fig. 1 represents the so-called "Hawk-Dove" game. As it is well-known, this game has two Nash equilibria in pure-strategy corresponding to the strategy profiles [C, D] and [D, C], thus yielding (5 ; 1) and (1 ; 5). There is also a mixed-strategy Nash equilibrium where each player plays C with a probability ½, yielding (5/2 ; 5/2). Now, suppose that both players are Bayesian rational and hold the following common belief regarding the probability that each strategy profile will be implemented (see fig. 2):

[5] In other words, the probability measure $\pi(.)$ in (1) corresponds to unconditional probabilities. The debate between *causal* and *evidential* decision theories concerns the conditions under which one is rationally allowed to use conditional probabilities. Ultimately, this debate is secondary because it is always possible to change the description of the decision problem such as to guarantee the independence of the probability distribution, even in the cases where there is a statistical and/or a causal relationship between acts and states of nature.

[6] It is worth insisting that this dependence is epistemic but not necessarily causal. The distinction is not always clearly entertained in the literature. For an insightful discussion of this point, see [14].

Fig. 2

		Bob	
		C	**D**
Ann	**C**	1/3	1/3
	D	1/3	0

From Ann's point of view, fig. 2 has the following interpretation: she believes that if she plays C, then there is a probability of ½ that Bob either play C or D; moreover, she believes that if she plays D, then Bob will play C for sure. It is easy to verify that Bob holds the same beliefs. These conjectures correspond to a strategy profile in mixed-strategy where each player plays C with probability 2/3. It appears clearly that given these beliefs, both players are maximizing their expected utility which confirms that they are Bayesian rational. Now, consider some probability space Γ and denote f: $\Gamma \rightarrow A$ some function mapping Γ onto a strategy profile a belonging to $A = A_{Ann} \times A_{Bob}$.[7] The function $f(.)$ implements a correlated equilibrium in the game corresponding to fig. 1 if and only if for each signal γ, $f(\gamma) = a$ is a strategy profile where all players are maximizing their expected utility conditional on the strategy they are playing. The corresponding numbers Prob$\{f^{-1}(a)\}$ define a correlated distribution over A. An instance of such correlated distribution is given by fig. 2 above.

Social scientists and philosophers have recently used Aumann's account as a foundation for a theory of rules and institutions ([7], [16], [17], [18], [19]). According to Gintis ([7], [16]) the correlating device represented by the function $f(.)$ plays the role of a "choreographer" who observes a random variable γ and then issues a directive $f(\gamma) \in A$. Gintis argues that social norms work along exactly the same kind of mechanisms: the norm literally signals to each player what he has to do in a given situation. A very similar point is made by Francesco Guala and Frank Hindriks ([17], [18]) as they argue that viewing rules as correlated equilibria help to overcome the traditional rules-versus-equilibria debate over the nature of institutions. Finally, it should be noted that Aumann himself, in a co-authored paper with Jacques Dreze [20], has used this account to characterize rational expectations in games. Therefore, if Bayesianism indeed justifies the use of the correlated equilibrium solution concept in game theory, then it seems that it might serve at least as a building block of some theory of social interactions. The rest of the paper discusses this possibility.

3. Bayesianism and the CPA: A Formal Framework

This section formally states Aumann's theorem on the basis of a *semantic epistemic model* (*s.e.m*). This will provide the basis to discuss the meaning of the CPA. A game G is a triple $< N, \{A_i, u_i\}_{i \in N} >$ with N the set of $n \geq 2$ players, A_i the set of pure strategies of player i and u_i player i's utility function representing i's preferences over the set of strategy profiles $A = \Pi_i A_i$. I assume that the players are Bayesian rational and thus the u_i functions have the property of uniqueness stated in the preceding section. A *s.e.m.* is a representation of the players' knowledge, beliefs and reasoning in a game G. It can also be called the *theory* of G. It corresponds to a structure $\mathcal{I}$: $< W, \{R_i, S_i, \pi_i\}_{i \in N} >$. W is the set of states of the world w that we assume to be finite. States of the world are similar to Savage's states of nature in the sense that a state (or "possible world") is a complete description of everything that is relevant for the

[7] Here, A_i = (C, D) for i = {Ann, Bob} and thus A = [(C, C), (C, D), (D, C), (D, D)].

modeler and the players. By definition, no uncertainty remains at a given state. An important difference however is that a state of the world w also includes the description of all players' choices while in Savage's framework the content of states of nature excludes the decision-maker's choice. As in the preceding section, $\pi_i(.)$ is a probability measure for player i over the state space W. I shall argue below that the interpretation of this measure in a game-theoretic framework is problematic from a Bayesian point of view. S_i is a function mapping any state w onto a strategy a_i, *i.e.* S_i: $W \rightarrow A_i$. S_i is thus player i's decision function. Finally, R_i is called an accessibility relation: it indicates which are the states w' that are epistemically accessible for player i from a given state w. Hence, wR_iw' means that w' is accessible from w for i. For convenience and because this is the standard practice in economics, I will not use the accessibility relations but instead the related possibility operators $\mathcal{P}_i$ and possibility sets P_i. The former corresponds to a mapping $\mathcal{P}_i$: $2^W \rightarrow 2^W$. Correspondingly, the possibility set P_iw is the set of the worlds that are accessible from w for i. It corresponds to the set of worlds that are indistinguishable for i or, in other words, all the worlds that i considers as possible at w. Thus, $\mathcal{P}_i(w) = P_iw$. The possibility set describes i's knowledge at w (what he rightly believes for sure) and the probability measure $\pi_{i,w}(.)$ determines i's beliefs at w. Since the players are Bayesian rational we set for any event $E \subseteq W$

(2) $$\pi_{i,w}(E) = \frac{\pi_i(E \cap P_iw)}{\pi_i(P_iw)}$$

In words, $\pi_{i,w}(.)$ is the probability i ascribes to the event E conditional on knowing P_iw.

The combination of a game G and a *s.e.m* $\mathcal{I}$ is called an epistemic game $\mathcal{G}$: < N, W, $\{A_i, u_i, R_i, S_i, \pi_i\}_{i \in N}$ >. What can be said about an epistemic game depends on the property of the *s.e.m* and especially of the accessibility relations R_i. Aumann's account follows the standard practice in information economics by assuming that each player's knowledge and beliefs correspond to an information partition I_i of the state space W. In terms of the possibility sets P_i, which implies the two following axioms:

(A1) $\forall w$: $w \in P_iw$.

(A2) $\forall w, w'$: if $P_iw \neq P_iw'$, then $P_iw \cap P_iw' = \varnothing$.

Axiom A1 states that the possibility set P_i represents i's knowledge defined as probability 1 true belief. Axiom A2 states that cells of the information partition are disjoint: if i considers w' possible but w'' impossible at w, then at w' he must consider w possible but w'' impossible. In other words, by A2 the sets P_iw are *equivalence classes*. As a last element of definition, we can now specify a set of knowledge operators K_i: $2^W \rightarrow 2^W$ where K_iE is to be read as "i knows the event E" and which we define as follows:

(K) $$K_iE = \{w \mid P_iw \subseteq E\}.$$

On this definition, i knows the event E at w if and only if all the worlds he takes as possible at w are members of E. We denote $\boldsymbol{S}(w) := (S_1(w), \ldots, S_n(w))$ the profile of correlated strategies where $S_i(w) = a_i$ for each player i. Correspondingly, $\boldsymbol{S}_{-i} := (S_1(w), \ldots, S_{i-1}(w), S_{i+1}(w), \ldots, S_n(w))$ We assume that the functions S_i are measurable with respect to the partitions I_i, *i.e.* they are constant over the partitions.[8] A player i is Bayesian rational at w if he maximizes his expected utility given his information I_i which corresponds to P_iw and $\pi_{i,w}$, therefore

[8] It is the same as assuming that i's knows which action he chooses.

(3) $E[u_i(\boldsymbol{S}) \mid I_i](w) \geq E[u_i(S_i', \boldsymbol{S}_{-i}) \mid I_i](w)$, for any $S_i' \neq S_i$ and E the expectation operator.

We are now ready to state Aumann's theorem.

Aumann's Theorem – Consider an epistemic game $\mathcal{G}$ with a *s.e.m.* $\mathcal{I}$ satisfying axioms A1 and A2 and where 1) each player is Bayesian rational at each *w* and 2) the players have a common prior, *i.e.* $\pi_1 = \pi_2 = \ldots = \pi_n = \pi$. Then, the strategy profile $\boldsymbol{S}$ is a correlated equilibrium distribution in the underlying game *G*.

Proof – Recall that a correlated distribution Prob$\{f^{-1}(a)\}$ is obtained on the basis of a function f: $\Gamma \rightarrow A$ mapping some probability space into a strategy profile. By setting $\Gamma = (W, \pi)$ we find that $\boldsymbol{S} := (S_1, \ldots, S_n)$ is such a function. Therefore, Prob$\{\boldsymbol{S}^{-1}(a)\} = \pi(w)$ is a correlated distribution in *G*. Moreover, since the players are Bayesian rational at all *w*, we know that they maximize their expected utility given their information. Therefore, $\boldsymbol{S}$ is an equilibrium in correlated strategies in *G*.

Aumann's theorem relies on four key assumptions made in the underlying *s.e.m.*. The first is that the players must have an information partition. As such, this is a strong and controversial assumptions because it depends on strong axioms regarding the epistemic abilities of the players (see below). There are arguments for and against the information partition assumption but this is beyond the subject of this paper. I will thus assume that it is unproblematic.[9] The second assumption is that the players are Bayesian rational at all states *w*. Given that the epistemic program consists in introducing Bayesian decision theory in a game-theoretic framework, this is not a particularly contentious one. Note however that it implies that Bayesian rationality is common knowledge (each player knows that each player knows that… everyone is Bayesian rational) and this may lead to some well-known "paradoxes of rationality" (*e.g.* [23]). I will also ignore this difficulty here. The third assumption is implicit but still an essential one to derive Aumann's result: it must be assumed that the players' choices and beliefs are causally independent. This directly follows from a difference between Savage's Bayesianism and Aumann's use of it in its account. In the former, the decision-maker's subjective beliefs range over features that are stochastically independent of his own choices and every causal relationship should be included in the description of the states of nature. In Aumann's treatment, since each player's beliefs range over the choices of other players this should imply that the state space differs from one player to another. The most obvious way to deal with this difficulty is to include each player *i*'s own choice in the description of the state, which is formally done through the decision functions S_i. This is not without consequences as it is no longer possible to distinguish acts from consequences as in Savage's SEUT as both are now part of the states' descriptions. This approach may lead to fallacious reasoning where one player's strategy choice seems to determine the other players' choices. We must thus impose an independence assumption in expression (3) according to which player *i*'s conjecture about $\boldsymbol{S}_{-i}$ is invariant across all *i*'s decision functions S_i.[10]

[9] The strongest argument in favor of the information partition assumption is that it is an almost natural one if we take an "external" perspective, *i.e.* the *s.e.m.* represents the point of view of the modeler but not necessarily of the players ([21], [22]).

[10] As demonstrated by [24], without this invariance assumption Bayesian rationality may conflict with causal rationality. As indicated above, it is worth noting that causal independence of choices and beliefs does not imply epistemic and thus stochastic independence.

The last assumption is the CPA. It states that the players agree over a probability distribution of the states of the world, which includes their choices. This assumption seems particularly contentious from a Bayesian point of view: from Savage's "personalistic" viewpoint, there seems to be no reason to expect an agreement over subjective beliefs. As a theory of social interactions, Aumann's account then seems to rely on an assumption that is unsustainable from a Bayesian point of view. I discuss these points in the next section, explicating Aumann's formalistic defense of the CPA.

4. Interpreting the CPA

The CPA is generally taken as the statement that differences in behavior or opinion are only due to differences in information. While it is essentially constitutive of information economics as a whole, the CPA reflects a metaphysical commitment with methodological and theoretical implications. In particular, it implies that coordination failures are uniquely the result of information differential. Therefore, this is not a benign assumption as it indicates to the social scientist where to look out to explain coordination successes or failures. At the same time, it is widely recognized that the CPA leads to theoretical puzzles and is only weakly supported empirically. On the theoretical side, the CPA leads to various forms of "no-trade theorems" [25]. However, these theoretical predictions are not supported by empirical evidence, particularly regarding the trading of financial assets: the fact that agents on the financial markets are willing to trade assets reflects a disagreement over the present value of the stream of future earnings generated by these assets. But, given the fact that the mutual willingness to trade reveals to each party some new information regarding the opinion of the other party, the CPA implies that no trade should take place. As Aumann has famously demonstrated [26], people with a common prior "cannot agree to disagree" when their opinions are common knowledge.

As argued by Stephen Morris [27], most of the justifications for the CPA are essentially pragmatic. In particular, it is sometimes suggested that giving up the CPA would imply an "anything goes" methodology where any economic phenomenon could be explained by the "right" heterogeneous priors. Alternatively, some scholars claim that it is more straightforward and/or relevant to capture heterogeneous priors as information processing errors or parameters in the utility function. These justifications are not convincing as they build on an arbitrary dichotomy between preferences and probabilities: while the latter are taken to be common to all agents, the former are generally allowed to be heterogeneous in economic models. But of course, allowing for preferences heterogeneity is no less *ad hoc* than allowing for probabilities heterogeneity. There is an alternative justification for the CPA that is more relevant here as it has been generally attributed to Aumann: the rational/logical justification. In a nutshell, it is the claim that the CPA follows from a property of rationality. Perfectly rational agents endowed with the same information must agree over their assessment of the likelihood of risky or uncertain events because there is no other basis for disagreement than levels of information and rationality. This view is generally referred as the "Harsanyi doctrine" as it has been exposed by John Harsanyi in his account of incomplete information games ([28], [29]). Harsanyi's point was partially theoretical: the CPA is theoretically convenient because it allows reducing any game of incomplete information to a game of imperfect information with "Nature" as an additional player. However, Harsanyi's claim was clearly a claim about the nature of rationality: in a world where all information is public, there is no ground to disagree about anything.[11]

[11] As an aside, it is interesting to note that Harsanyi developed an almost parallel argument in his writings on Utilitarianism regarding the "extended preferences" people would have behind a veil of ignorance. Here, Harsanyi argued that ultimately rationality entails agreement over preferences if people do not have any private information regarding their personal identity. For a discussion of this point, see [30].

Indeed, Harsanyi [31] even argued that Bayesianism does not imply radical subjectivism but is compatible with a form of “necessitarianism” which “uniquely specifies the subjective probabilities which a rational decision-maker can use in a given situation” (p.120). Accordingly, he proposed several criteria (simplicity, indifference principle…) to rationally determine objective priors.

However, despite Harsanyi’s claim to the contrary, the rational/logical justification of the CPA fits uneasily with Bayesianism. It may be true that there are some situations where it is possible to agree over a set of objective probabilities. For instance, if I buy a lottery ticket I may compute my chances to win and I may reasonably be able to convince you that my estimation reflects the objective probability of winning. Rational agreement over objective probabilities seems even more straightforward in the case of natural events occurring on a repetitive basis. In this latter case, it may be argued that the (rational) Bayesian assessment of the likelihood of natural events should converge to the frequentist assessment. It is worth noting that the case for a rational agreement may be disputed even in the preceding examples. In the lottery ticket example, my assessment of the winning probabilities depends on a subjective belief about the fairness of the lottery which may differ from your subjective belief. Regarding natural events, any probabilistic assessment based on repetitive occurrences depends on a tacit belief about the stability of the underlying causal structure of the world (or more generally of its metaphysical features). Arguably, believing in such stability may be indeed reasonable or even “rational” (based on Bayes’ rule) but the point remains that subjective assessments should converge only under highly specific conditions.

In any case, what may be true for lotteries or natural events is not true for most social and cultural events. The reason is well-captured by game-theoretic models based on the framework of the preceding section: the occurrence of social events is a function of people’s behaviors and the latter are a function of each individual’s beliefs and rationality. In other words, the “correct” or “objective” beliefs about these events depend on the subjective beliefs held by each member of the population about these events, and thus on subjective beliefs about these beliefs and so on. As it appears, social events are constituted by *infinite belief hierarchies* and the beliefs about these events are already part of these belief hierarchies. Now, suppose that there is a set φ of criteria that we deem as relevant for fixing “rational” priors over social events. Call a person who uses these criteria φ-Bayesian. Suppose that Ann uses these criteria to determine her prior (she is φ-Bayesian); obviously, her “correct” prior depends on Bob’s decisions which themselves are function of Bob’s prior belief. Can Ann assume that Bob will determine his prior on the basis of φ? Clearly, this must be the subject of a *subjective* Bayesian assessment. If Ann gives a probability one credence to the event that Bob is φ-Bayesian, then the criteria φ may indeed be rationally used by Ann to determine the correct probabilities, even though she may ultimately be wrong. But if Ann does not believe that Bob is φ-Bayesian, then she should not use the φ criteria because she cannot expect to converge toward the same probabilistic assessment than Bob on their basis. The conclusion is therefore that as far as social events are concerned, a “necessitarian” view of subjective beliefs is mistaken unless everyone comes to share and to agree over this necessitarian view.

A straightforward implication of the preceding argument is that the CPA lies outside the realm of strict Bayesianism. This is indeed what Gintis [7] claims when he writes that “[c]ommon priors… are the product of common culture” (p.141). Social norms play the role of a choreographer precisely through implementing a common prior probability distribution over the space of social events. Similarly, it is plausible to see the existence of a common prior as the result of community membership [9]. The fact that a set of individuals belong to the same community, share a set of features including a common history and that they know this fact may be sufficient to ground the existence of a common prior. Indeed, this point can be captured on the basis of a *s.e.m.* similar to the one presented in section 3. Community membership may be viewed as an event $E \subseteq W$ such that for any $w \in E$, $\pi_{1,w} = \pi_{2,w} = \ldots = \pi_{n,w} = \pi_w$, while allowing for heterogeneous priors over $W\backslash E$. Now, on the basis of the individual possibility operators $\mathcal{P}_i$, define the *common possibility operator* $\mathcal{P}^*$ as follows:

(CP) $\mathcal{P}^*(w) = \bigcup_{k=1}^{\infty} P_k^* w$,

with $P_1^* w = \bigcup_{i \in N} P_i w$ and $P_k^* w = \bigcup\{P_1^* w' | w' \in P_{k-1}^* w\}$.

In words, the common possibility operator maps any state of the world w into a set corresponding to the transitive closure of the players' possibility sets at w. Denote the resulting *common possibility set* P^*w. As a consequence, the common possibility set can be associated to a common knowledge operator K^* that is obtained in the standard way:

(CK) $K^*E = \{w \mid P^*w \subseteq E\}$.

According to (CK), an event is common knowledge among a set of players at w if and only if all the worlds in the communal possibility set are members of E.[12] It is easy to check that $\mathcal{P}^*$ and P^* have exactly the same properties as their individual counterparts (especially axioms A1 and A2). As a result, they define a common partition $\boldsymbol{I}$ of the space W. Suppose now that the event E above that everyone is a member of the same community is such that K^*E, *i.e.* is common knowledge among the n players. That means that E corresponds to an atom of the partition $\boldsymbol{I}$. This atom is thus a subset of W where, by assumption, the players have a common prior over the subspace $(W - W\backslash E) = W^E$. We can then construct a new *s.e.m.* $\mathcal{I}^E$ where W^E is substituted for W as the relevant state space while the CPA is now holding. By Aumann's theorem, the resulting correlated strategy profile $\boldsymbol{S^E}$ is a correlated equilibrium with the corresponding distribution $\pi^E = \pi_w$. On this interpretation, community membership is constitutive of the CPA: the fact that it is commonly known that everyone belongs to the same community warrants the agreement over the probabilistic assessment of social events that Harsanyi was seeking to ground as a property of Bayesian rationality. Of course, this interpretation does not completely settle the whole issue as one may rightly ask how the sole common knowledge of community membership may trigger such a strong epistemic agreement. Moreover, common knowledge of community membership may itself be regarded as too strong an assumption on the ground that there are few if any "public events" in the social world that may lead to such an epistemic state.[13] Social scientists and philosophers disagree on this matter, some arguing that public events are inexistent (*e.g.* [35]) while others see them as being foundational for human societies (*e.g.* [36]; [37]).

[12] Note that this definition of common knowledge relies on the tacit assumption that each player "knows" the other players' possibility operators (and thus their information partitions), where knowledge here refers to an informal notion (*i.e.* not define by the knowledge operator). This assumption itself may be interpreted in different ways. A first possibility is to give it a substantive meaning in terms of "mutually accessible natural occurrences" ([7], [32]). Another possibility is to regard this assumption as a mere formal artefact that results from the mathematical framework adopted. This is clearly the interpretation intended by Aumann [33]. Obviously, the issue is deeply related to the interpretation of the CPA and to the fact that those priors (common or not) are necessarily commonly known in any *s.e.m.*. See below.

[13] A public event is an event E such that $E = K_iE$ for all $i \in N$. First, note that this entails $P_iw \subseteq E$ for all $i \in N$ and all $w \in E$ and thus $E = K^*E$. Now consider any event F such that K^*F. Then, it can be shown that there exists a public event $E \subseteq F$. Indeed, $E \subseteq F$ implies $K^*E \subseteq K^*F$ and thus $E \subseteq K^*F$. In other words, common knowledge events depend on the existence of public events. See the related discussion of evident knowledge in [34].

This is not to place to settle this dispute. First and as indicated above, endorsing the view that the CPA is grounded on the existence of community and social norms takes us outside the territory of Bayesianism and of the epistemic program. Of course, this is not a problem *per se* but since I am interested in evaluating the possibility for Bayesianism to provide a theory of social interactions, this view is not satisfactory. A second reason lies in the fact that the above interpretation *is not* the one retained by the main proponent of the CPA, Robert Aumann [38]. Actually, Aumann presents a quite different and purely formal defense of the CPA that he intends to be more compatible with Bayesianism. I shall argue however that this defense is meaningless as a part as a theory of social interactions and that it even casts doubts over the relevance of the notion of prior beliefs in a game-theoretic context. To understand this defense, it is useful to note that it takes place as a response to previous critiques of the CPA, especially Faruk Gul's one [39]. Gul argues that the CPA is a "philosophical position" and that its status depends on the interpretation one has of the *s.e.m.*: the "prior view" or the "hierarchy representation". On the former, "we imagine a situation prior to the economic problem considered" (p.923). The priors π_i reflect the (common knowledge) players' beliefs at this prior stage. A state of the world w is realized at a subsequent stage where each player receives an information, updates his prior $p_{i,w}$ and adjusts his behavior $S_i(w)$ accordingly. Crucially, "[i]n this interpretation, the prior stage represents a situation which actually occurred at some previous time" (p.924). On the latter, each player is endowed with a "type" t_i taken from a set $T = \prod_{i \in N} T_i$. A type specifies the player's strategy choice as well as his belief over the types of other players. Since a belief about others' types implies to have a belief over others' beliefs about each player's type, a type specifies the player's infinite belief hierarchy. It follows that the type profile $t := (t_1, .., t_n)$ in a *s.e.m.* is a complete specification of some infinite belief hierarchies. Contrary to the preceding interpretation, it is not assumed that there is a prior stage. The belief hierarchies are given once and for all in a purely static fashion.

According to Gul, the prior beliefs have no meaning in the hierarchy interpretation: "The prior are artifacts of a notational device to represent the infinite hierarchies of beliefs on X [the set of parameters including players' strategy choices] of the players, *i.e.* their "posteriors" at the true state of nature" (p.925). As a result, Aumann's theorem has no obvious interpretation. Quite the contrary, prior beliefs do have meaning in the prior view as they correspond to beliefs actually held at some definite point of time in a dynamic process. However, this leads to two difficulties: firstly, it is not clear what is implied by the fact that each player has a belief over his own strategy choice as it seems to recover the difficulties with the traditional interpretation of mixed strategies. Secondly, assuming that there is an *actual* prior stage, it is of course not a necessity that the players have common knowledge of their priors or that these priors have to be identical. However, as the theory $\mathcal{T}$ is the *s.e.m.*, it is necessarily true at all states $w \in W$. As the players' priors are assumed to be constant across W, they correspond to theorems (or "tautologies") in the corresponding syntax and thus have to be (commonly) known by the players.[14] As a result, if we retain the prior view, Aumann' theorem while interpretable seems to rely on a very strong and hardly empirically justifiable epistemic assumption.

The fact that prior beliefs have no clear meaning in the hierarchy interpretation has been recognized by others. For instance, Brandenburger and Dekel [40] note that "it is the conditionals, and not the priors, that are of decision-theoretic significance". Still, the same authors have established that these two interpretations correspond to completely isomorphic mathematical constructions. Indeed, this is easily seen once it is understood that a state of the world w is equivalent to a type profile $(t_1, .., t_n)$ possibly combined with an atom z of some

[14] The same is obviously true for the players' information partitions. See footnote 13 above for the peculiar meaning of this common knowledge.

uncertainty space Z.[15] This suggests that either (as Gul argues) the prior view and the hierarchy interpretations refer to a dynamic social situation and a static social situation respectively or that the priors have no substantive (social-theoretic) meaning. Judging by Aumann's defense of CPA, the latter is the most likely. In [39], he explicitly formalizes a dynamic model where the one of the players learn some new information.[16] A dynamic model is consistent if it satisfies two axioms: a) players with the same information have the same probabilities and b) probabilities are updated by Bayes' rules when new information is acquired. What he establishes is the following:

If a dynamic model is consistent, then the players have a common prior.

The result is relatively transparent as the first axiom is basically an informal statement of the CPA. The way Aumann interprets this result is more interesting. First, he notes that the prior stage (the first period in the model, before one of the players receives an information) need not be actual but can (and most of the times will) be *hypothetical*. This makes this dynamic model equally relevant for the prior view and the hierarchy interpretation. The fact that there is an actual prior stage is irrelevant, as it is a feature of any axiomatic system that "the arguments depend crucially on hypothetical, artificial situations that never existed" (p.935). Arguably, an axiom is nothing but a statement about what would result if some conditions hold. There is clearly no reason to expect that it must necessarily have an empirical counterpart. It follows that the meaning or the interpretation of the CPA does not depend on the existence of an actual prior stage. If such an actual prior stage exists, then it is true that the (common) knowledge of the (common) players' prior is not a tautology. But the previous theorem establishes that this must be due to differential information and that in this case there must be in principle a hypothetical stage preceding the actual prior stage where the players have a commonly known common prior.[17]

Aumann's defense is thus merely formal and even tautological. It consists in insisting that if only differential information explains differences in probabilities, then Bayesian rationality implies the CPA. But of course, assuming that heterogeneous probabilities are only due to differential information is already stating the CPA. This will hardly convince anyone not already convinced by the CPA and the most charitable way to interpret Aumann's defense is as a claim about the nature of the economic methodology: economists are methodologically committed to explain behavior in terms of information and though not the only methodological approach available, economists are justified to do so for some specific reasons. Obviously, this defense fails as far as Bayesianism is concerned: the CPA has simply nothing to do with Bayesianism. It also has a disturbing implication regarding the meaning of *s.e.m.* as a whole: on Aumann's defense, the fact that the common prior and more generally the theory $\mathcal{J}$ are (commonly) known is devoid of any substantive meaning: it is merely a mathematical artefact that results from the nature of the semantic model used. Indeed, the *s.e.m.* "is not a "model" in the sense of being exogenously given; it is merely a language for discussing the situation. There is nothing about the real world that must be commonly known among the players" ([4], p.1177). This is coherent with the formal understanding of the CPA: the *s.e.m.* does not correspond to

[15] In this case, we have $W = T \times Z$.

[16] As this necessitates further notations, I present Aumann's model in the appendix at the end of this paper.

[17] Aumann rejects the case of commonly known heterogeneous priors on the ground that there is "no evidence or even argument that such a situation is tenable" (p.934). This remark is slightly surprising given the fact that it is probably relatively easy to find people endowed with the same information and nonetheless disagreeing on some issue. The existence of betting markets for sport competitions provides a great example. Aumann's remark only makes sense in the context of his very model where indeed, such commonly known heterogeneous priors are ruled out by the axioms.

propositions that are known by the players themselves; it is rather a set of propositions used by the modeler to describe and analyze some (virtual or concrete) situation. Thus, even though a *s.e.m.* is a tool to study how individuals reason in strategy interactions, it does not pretend to describe how they actually reason or even how they should reason.

5. Symmetric Reasoning in Strategic Interactions

If Aumann's defense has been correctly interpreted, then it is hardly a satisfactory one if Bayesianism is to be conceived as a theory of social interactions. Still, the most likely conclusion is that Bayesianism in a game-theoretic context cannot lead to firm predictions about or convincing explanation of people's behavior without adding other theoretical assumptions and principles.[18] However, a more problematic implication of Aumann's defense concerns the very notion of prior belief. The latter refers to nothing that could correspond to the players' actual beliefs. The probability measures π_i are simply meaningless from a social-theoretic point of view.[19] In this section, I present an alternative account of the players' ability to coordinate which gives up any reference to prior beliefs. Instead, I build on the notion of *symmetric reasoning* which finds its roots' in David Lewis' theory of conventions [42]. However, I retain the assumption that the players' maximize their expected utility on the basis of beliefs that are directly derived from actual reasoning processes.

Lewis is one of the first scholars to have developed a theory of common knowledge. This was needed in Lewis' account since he required for a behavioral regularity to count as a convention that it is common knowledge in the relevant population. Though Lewis' informal account of common knowledge is often compared to more formal treatments such as the one proposed by Aumann [26] (on which the above definition of common knowledge is based), it actually adopts a quite different perspective [43]. In the *s.e.m.* of the preceding sections, common knowledge of some event corresponds to an event which may hold or not. Lewis' goal was to specify the conditions under which common knowledge obtains. More exactly, the point was to characterize how common knowledge is generated in a population. Lewis' account is actually stated not in terms of knowledge but rather in terms of the weaker notion of "reason to believe" and relies on the key notion of *indication*. It is formulated as follows [42, p.52-3]:

> "Take a simple case of coordination by agreement. Suppose the following state of affairs – call it *A* – holds: you and I have met, we have been talking together, you must leave before our business is done; so you say you will return to the same place tomorrow. Imagine the case. Clearly I will expect you will return. You will expect me to expect you to return. I will expect you to expect me to expect you to return. (…)
>
> What is about *A* that explains the generation of these higher-order expectations? I suggest the reason is that *A* meets these three conditions:
>
> (1) You and I have reason to believe that *A* holds.
> (2) *A* indicates to both of us that you and I have reason to believe that *A* holds.
> (3) *A* indicates to both of us that you will return."

On this basis, Lewis showed that, provided that "suitable ancillary premises regarding our rationality, inductive standards, and background information" are satisfied, the satisfaction of

[18] Of course, it is still possible to retain the analysis in terms of *s.e.m.* while allowing heterogeneous priors. In this case, the framework of section 3 leads to the solution concept of *subjective* correlated equilibrium. As Aumann [6] points out however, this concept places very few restrictions over the players' behaviors and beliefs.
[19] Some philosophers and game theorists are explicit on this, *e.g.* [41].

these three conditions entails common reason to believe in the fact that you will return. Lewis' point is thus that a mutually known or believed proposition x is sufficient to entail the common knowledge or belief of another proposition y if a) the mutual knowledge (belief) of x indicates a second-order knowledge (belief) of x, b) x indicates y to each member of the population, and c) it is common knowledge (belief) that x indicates y to each member of the population. The last proviso is essential: I cannot infer from my beliefs of x and that you believe x and from the fact that I infer y from x that you also believe y, unless I also believe that you infer y from x. Now, to obtain common knowledge (belief) of y, I need to infer from my knowledge (belief) of n-order mutual knowledge (belief) of x a n-order mutual knowledge of y, which requires a knowledge (belief) that x indicates y to each of us for all n ([9], [44]).

This result may strike as being fairly similar to the definition of common knowledge in terms of public events in *s.e.m.* with information partitions. Indeed, in such models, for any common knowledge event F there is a public event E. A public event is defined as an event that is necessarily mutually known, *i.e.* $\mathrm{E} \subseteq K_iE$ for all i. Then, this public event implies the event that F is common knowledge, *i.e.* $\mathrm{E} \subseteq K^*F$.[20] There is a key difference however lying in Lewis' use of the notion of *indication*. In *s.e.m.* events are related by the inclusion operator $\subseteq$ which formalizes by definition a relation of *logical implication*. However, the indication relation does not reduce to logical implications. It also refers to all inductive standards that may be used in practical and theoretical reasoning. To take a trivial example, the fact that all drivers stop at red lights but do not stop at green ones has nothing to do with logic. More exactly, what allows me to infer from my knowledge that the light is red that I should stop is not a logical implication. It is rather the result of a combination of some basic principle of practical rationality (I do not want to have a car accident) with an inductive standard according to which a red light indicates to me that cars from my right and left will not stop. Note moreover that this indication holds only if I assume that other drivers use the same inductive standard. Of course, this is due to the fact that this example is a social event.

In the rest of this section, I sketch a formal account encompassing Lewis' notion of indication. I do not claim any originality in this. My point however is elsewhere: I show that the fact that players are symmetric reasoners (they use the same inductive standards) leads to *s.e.m* formally identical to those incorporating the CPA. To prove this, I need however to take a different approach as the use of a *s.e.m.* cannot capture the specificity of the indication relation. As I note above, *s.e.m.* reduce all relations between events to inclusion operations and thus to logical implications. Therefore, instead of proceeding directly at the semantic level, I start by specifying a syntax. This allows me to express the indication relation and its properties formally. A syntax is a formal language that build on an alphabet $\mathscr{A}$ that consists here in the following elements: 1) a set of atomic propositions p, q, r… corresponding to sentences; 2) the logical connectives $\neg$ ("not"), $\wedge$ ("and"), $\vee$ ("or") and $\rightarrow$ ("if… then"); 3) a set of n probabilistic belief operators B_1^π, …, B_n^π where $B_i^\pi p$ corresponds to the sentence "i believes proposition p is true with probability of degree π". I use belief operators instead of knowledge operators for two reasons: first, as we have seen above, Lewis reasoned in terms of "reason to believe" rather than knowledge; second, reasoning in terms of knowledge would make necessary to define a probability measure to characterize beliefs. Here, the use of probabilistic operators allows ascribing directly degrees of beliefs to propositions without any direct reference to such a probability measure.

A *formula* is a finite string of symbols formed by combining connectives and probabilistic operators from the atomic propositions. The point here is to define a list $\mathrm{F_V}$ of valid formulae

[20] See footnote 14.

(or *theorems*) that correspond to some epistemic game $\mathfrak{G}$. Thus, we need a language sufficiently rich to characterize the constitutive features of the corresponding game G as well as those of the related *s.e.m.* $\mathcal{I}$. In particular, we need a set of propositions expressing what the players are doing (their strategy choice), a set of axioms satisfied by the probabilistic operators B_i^π and a set of axioms charactering the indication relation. Regarding the first element, I simply write D_ix for "i plays x" where x is any strategy in the corresponding game. Regarding the probabilistic operators, I will assume that it satisfies the following axioms:[21]

- **N**: $\forall p \in \mathrm{F_V}$: $B_i^1 p$.
- **K**: $B_i^{\pi 1} p \wedge B_i^{\pi 2}(p \rightarrow q) \rightarrow B_i^{\pi 3} q$, with $\pi 3 = \pi 1 \mathrm{x} \pi 2$.
- **D**: $B_i^\pi p \rightarrow B_i^{1-\pi} \neg p$.
- **PI**: $B_i^\pi p \rightarrow B_i^1 B_i^\pi p$.
- **NI**: $\neg B_i^\pi p \rightarrow B_i^1 \neg B_i^\pi p$.

Axiom N states that all theorems (*i.e.* necessarily true formulae) are believed with probability 1 by the players. Axiom K indicates that if one believes some proposition p with probability $\pi 1$ and believes with probability $\pi 2$ that p logically implies q, then he believes q with probability $\pi 3$. Axiom D requires that one's beliefs be consistent. It implies in particular that if I believe with probability 1 that p is true, then I cannot believe with any strictly positive probability that $\neg p$ is true (p is false). Axioms PI (positive introspection) and NI (negative introspection) state that one believes with probability 1 that he has the beliefs he has and that he does not have the beliefs he does not have. They guarantee that the players have information partitions in the corresponding *s.e.m.*. It is worth noting that since we are dealing with belief operators, I have not specified any "knowledge axiom" which would entail that beliefs are necessarily true even when they are with probability 1.

I can now characterize the indication relation. In this framework, I will take the sentence "p indicates q to i" to mean that if i believes that p is true with probability $\pi > ½$, then he also believes that q is true with probability $f(\pi) > ½$. It is natural to restrict the definition of the indication relation to beliefs of degree $\pi > ½$ since given axiom D, $B_i^\pi p$ with $\pi < ½$ entails $B_i^{1-\pi} \neg p$.[22] The function $f(.)$ captures the particular kind of inductive reasoning that i is using to infer q from p.[23] I will allow for the possibility that each player has at his disposal several modes of reasoning including the deductive one. Denote R the set of such reasoning modes and r any specific mode. I will assume that the indication relation $\underset{r,i}{\Rightarrow}$ satisfies the following properties for all $r \in R$:

(I1) $B_i^\pi p \wedge (p \underset{r,i}{\Rightarrow} q) \longrightarrow B_i^{f_r(\pi)} q$.

(I2) $(p \longrightarrow q) \longrightarrow (p \underset{r,i}{\Rightarrow} q) \longrightarrow (B_i^\pi p \longrightarrow B_i^{f_r(\pi)} q)$ with $f_{r,}(\pi) = \pi$, for all $r \in R$.

(I3) $\forall i, j \in N$: $(B_i^\pi p \longrightarrow B_i^{f_r(\pi)}(B_j^{\pi'} q)) \wedge B_i^\pi(B_j^{\pi'} q \longrightarrow B_j^{f_r(\pi')} t) \longrightarrow (B_i^\pi p \longrightarrow B_i^{\pi f_r(\pi)}(B_j^{f_r(\pi')} t))$.

(I4) $(p \underset{r,i}{\Rightarrow} q \wedge q \underset{r,i}{\Rightarrow} t) \longrightarrow p \underset{r,i}{\Rightarrow} t$ with $B_i^{f_r(\pi)} q = B_i^{f_r(\pi)} t$.

[21] They are similar to the axioms of the modal logic KD45 which are generally used to characterize belief operators. The small differences are due to the fact that we are using probabilistic operators.
[22] We may consider that $\pi = ½$ corresponds to the limit case where one is agnostic about the truthiness of some proposition. Though it could be dealt with, I will simply assume that true agnosticism is impossible.
[23] See [45] for a more detailed discussion of this account of Lewis' indication relation.

The notation is slightly cumbersome but the properties are relatively straightforward. Axiom I1 actually corresponds to the informal definition of the indication relation I have given above: if i believes p with probability π then, according to some mode of reasoning r, he believes q with probability $f_{r,i}(\pi)$. Axiom (I2) reflects the fact that the indication relation does not contradict the relation of logical implication. In this case, the reasoning mode r corresponds to the deductive mode: if p logically implies q and i believes p with probability π, then logical reasoning implies that he must believe q with probability π. Note that I2 follows from axiom N above. Axiom I4 states that the indication relation satisfies a property of transitivity. Finally, Axiom I3 says that if under reasoning mode r, p indicates to i that j believes q with probability π' and if i believes that under r, q indicates t to j, then *under reasoning mode r*, p must indicates to i that j believes t with probability $f_{r,j}(\pi')$. Note that it is implicitly assumed that i attributes to j the same reasoning mode r than he is using. This is captured by the expression $B_i^{\pi}(B_j^{\pi'} q \longrightarrow B_j^{f_r(\pi')} t)$ which should be read as "i believes that under reasoning mode r, q indicates t to j *and that j is actually reasoning according to r*".

It is possible to dispense with this last clause if we assume from the start that players are symmetric reasoners, *i.e.* they use the same reasoning mode r and they believe this with a sufficient probability. This is captured by the following axiom:

(SR) $(p \underset{r,i}{\Rightarrow} q) \longrightarrow B_i^1(p \underset{r,j}{\Rightarrow} q) \longrightarrow B_i^1(B_j^{\pi'} p \longrightarrow B_j^{f_r(\pi')} q)$

We may now use these axioms to formalize Lewis' account of the generation of common belief in a population. In our notation, Lewis' three conditions correspond to the following expressions ($B_N^{\pi}p$ denotes that p is mutually believed with probability π in N):

(L1) $B_N^{\pi} p$.

(L2) $(p \underset{r,N}{\Rightarrow} B_N^{\pi} p) \longrightarrow (B_N^{\pi} p \longrightarrow B_N^{f_r(\pi)} B_N^{\pi} p)$.

(L3) $(p \underset{r,N}{\Rightarrow} q) \longrightarrow (B_N^{\pi} p \longrightarrow B_N^{f_r(\pi)} q)$

L1, L2 and L3, combined with SR, entail that q is commonly believed in N. To see this, note that combining L2 and L3 with I3 and SR entails[24]

(L4) $B_N^{\pi} p \longrightarrow B_N^{f_r(\pi)} B_N^{f_r(\pi)} q$.

Combining L2, L4, I3 and SR leads to

(L5) $B_N^{\pi} p \longrightarrow B_N^{f_r(\pi)} B_N^{f_r(\pi)} B_N^{f_r(\pi)} q$.

And so on. Moreover, combining L1 and L3 on the basis of I1 gives

(L3') $B_N^{f_r(\pi)} q$.

Similarly, combining L1 and L4 leads to

(L4') $B_N^{f_r(\pi)} B_N^{f_r(\pi)} q$.

[24] First combine L3 with SR, which gives $B_N^1(B_N^{\pi} p \rightarrow B_N^{f_r(\pi)} q)$. Combine this results with L2 and I3 to obtain $B_N^{\pi} p \longrightarrow B_N^{f_r(\pi)} B_N^{f_r(\pi)} q$.

And so on. As we may continue indefinitely, this proves that q is commonly believe to degree $f_r(\pi)$ in the population.

Consider the case where proposition q is a sentence corresponding to the conjunction $D_1x \wedge D_2x \wedge \ldots \wedge D_nx$ and proposition p is a description of any specific *game situation*. The following result may be established:

Theorem 1: Symmetric reasoning in games – For any game G, consider a syntax with an alphabet $\mathscr{A}$ and the list of axioms ({N, K, D, PI, NI,}, {I1, I2, I3, I4, SR}). Suppose that there is at least one proposition p describing a game situation that satisfies conditions L1 and L2, and suppose that there is at least one corresponding proposition ($D_1x \wedge D_2x \wedge \ldots \wedge D_nx$) that satisfies condition L3 and such that each player maximizes expected utility. Determine on this basis the set of theorems F_V. Then, we can construct a *s.e.m.* $\mathscr{I}$: $< W, \{R_i, S_i, \pi_i\}_{i\in N} >$ satisfying axiom A2, Bayesian rationality and the CPA.

Proof – See the appendix.

The significance of this result lies in the fact that it shows that a modified version of Aumann's theorem can be recovered without endowing the players with any form of prior belief over their own and others' actions. Intuitively, symmetric reasoning is substituted for the CPA. The correlation of the players' behavior is achieved through an assumption regarding their theoretical and practical rationality while eschewing the difficulties related to the interpretation of the CPA and the very notion of prior beliefs. Naturally, the symmetric reasoning assumption is a strong one and its empirical significance has to be established. The notion of community-membership briefly surveyed in section 4 may provide an explanation for the fact that individuals are symmetric reasoners in some cases and not in others.

6. Conclusion

The CPA is essential to Aumann's claim that the concept of correlated equilibrium is the expression of Bayesian rationality. However, it cannot be defended on the ground of Bayesianism alone. Without it, very few restrictions can be set up regarding the rational way to play games. As far as Bayesianism and the epistemic program in game theory aim to constitute a philosophy of social interactions, this is arguably problematic. Aumann's own defense of the CPA does nothing to remedy this problem as it is purely formal. It is based on a metaphysical principle according to which differences in behavior and opinions are due to differences in information. Moreover, it makes the very notion of prior beliefs difficult to interpret.

However, if one is ready to step outside Bayesianism, there are ways either to ground the CPA on a social theory or to substitute assumptions over the players' reasoning modes for it. In the former case, the fact that the players agree over a common prior may be seen as the product of community-membership. In the latter, social coordination is achieved through the fact that players are symmetric reasoners with respect to some game situations. Community-membership may obviously play a role in fostering symmetric reasoning in a population.

Appendix

A – Aumann's Dynamic Model

This section presents Aumann's formal defense of the CPA that is discussed informally in section 4. It builds on the demonstration that a consistent dynamic model (in some well-defined sense) entails the CPA.

Consider a *s.em.* $\mathcal{J}$ satisfying axioms A1 and A2. Therefore, each player has an information partition I_i over the state space W. We call $I = (I_1, \ldots, I_n)$ the resulting *partition profile* and $\boldsymbol{P}$ the corresponding partition structure. A *dynamic framework* is a family $\mathcal{F}$ of partition structures $\boldsymbol{P}$ and thus of *s.e.m.* with the same state space W and the same set of players N, that is closed under coarsening:

(0) For any partition profile I with structure $\boldsymbol{P} \in \mathcal{F}$, a profile I' can be obtained by coarsening one or several I_i, *i.e.* the atoms in I' are the union of atoms of I. Then, there is a structure $\boldsymbol{Q}$ in $\mathcal{F}$ with the profile I'.

Intuitively, the structure $\boldsymbol{Q}$ and $\boldsymbol{P}$ represent two patterns regarding the distribution of information and where information is more important in the latter, *i.e.* everything that is known in $\boldsymbol{Q}$ is known in $\boldsymbol{P}$ but not the converse. The dynamic framework $\mathcal{F}$ is a model of information acquisition as it shows how partition structures change as new information arrives. The model is consistent if for all $\boldsymbol{P}$, $\boldsymbol{Q} \in \mathcal{F}$, all events E and all players i, j the following axioms obtain:

(1) If E is an atom of both I_i and I_j in $\boldsymbol{P}$, then $\pi_{i,E}^{P}(.) = \pi_{j,E}^{P}(.)$, where $\pi_{i,E}^{P}(.)$ is i's conditional belief at E in structure $\boldsymbol{P}$.

(2) Assume that I_i in $\boldsymbol{P}$ is a refinement of I_i' in $\boldsymbol{Q}$ and $I_j = I_j'$ for all other j. Define A as an atom of I_i and B as an atom of I_i' with $\mathrm{A} \subseteq \mathrm{B}$. Then, $\pi_{i,A}^{P}(E) = \frac{\pi_{i,B}^{Q}(\mathrm{E} \cap \mathrm{A})}{\pi_{i,B}^{Q}(\mathrm{A})}$ and $\pi_{j,w}^{P}(.) = \pi_{j,w}^{Q}(.)$ for all $j \neq i$.

The first axiom is a formal statement of the idea that if the players know "nothing" (*i.e.* they do not have any private information), then they must have the same probabilities. The second axiom simply states that when information is acquired, the players' update their probabilities on the basis of Bayes' law.

Now, a theorem can easily be established:

Theorem – If F satisfies (0), (1) and (2), then $\pi_1^P(.) = \ldots = \pi_n^P(.) = \pi^P(.)$ for all $\boldsymbol{P} \in \mathcal{F}$.

In words, a consistent dynamic framework implies the CPA. To see this, take $i = 1, 2$, fix $E = W$ in (1) and denote $\boldsymbol{Q}$ and I^Q the corresponding partition structure and partition profile. Denote $\boldsymbol{P}$ the structure obtained by refining *both* information partitions in I^Q, *i.e.* I_1^Q and I_2^Q are coarsening of I_1^P and I_2^P respectively. Finally, we write $I^R = (I_1^P, I_2^Q)$ the "intermediary profile" corresponding to the case where 1 has acquired some private information but not 2. Since I^R is a coarsening of I^P, by (0) the profile I^R corresponds to a partition structure $\boldsymbol{R} \in \mathcal{F}$. (1) entails

$\pi^{Q}_{1,W}(.) = \pi^{Q}_{2,W}(.)$ in Q. Then, by passing from $\boldsymbol{Q}$ to $\boldsymbol{R}$ and then from $\boldsymbol{R}$ to $\boldsymbol{P}$ using (2), we find that $\pi^{P}_{i,A}(F) = \frac{\pi^{Q}_{i,W}(F\cap A)}{\pi^{Q}_{i,W}(A)}$ for any event *F*. Hence, players 1 and 2's conditional probabilities differ if and only if the atoms *A* in their information partitions differ, *i.e.* if they do not have the same information.

B – Symmetric Reasoning in Games

In this section, I sketch a proof for Theorem 1 which is given in section 5. What has to be established is that there exists at least one *s.e.m.* corresponding to the syntax described in section 5 and that among these *s.e.m.*, there is at least one that has the properties given in the main text, especially the fact that the players have a common prior over some state space. I do not try to show that any *s.e.m.* with a common prior entails symmetric reasoning, though this could be demonstrated (which entails that the CPA and symmetric reasoning are actually equivalent). I will provide the demonstration by means of an example of a 2-player-2-strategy game. This is without loss of generality at least as long as we assume that the number of players and strategies is finite.

Consider a game *G* with *N* = (1, 2), *A* = ((*x*, *y*), (*v*, *z*)) and where both players have a utility function u_i, *i* = 1, 2, satisfying Savage's axioms. Throughout, I assume that the players commonly believe with degree 1 that they are playing this game. First, we construct a syntax to describe how the players play this game. We use an alphabet $\mathscr{A}$ that contains the standard logical connectives and the atomic propositions *p*, *q*, *x*, *y*, *v*, *z*. The first two propositions will be used to refer to "game situations", *i.e.* specific situations in which the players are and on the basis of which they can make decisions. For instance, *p* may denote "the traffic light is green" and *q* "the traffic light is red". The propositions *x*, *y* ,*w* and *z* simply refer to the strategy choices of the players. Finally, we need the formulae D_1x, D_1y, D_2v and D_2z to describe what the players are doing. We define the probabilistic belief operators B_i^π with $B_i^\pi p$ the formula meaning "player *i* believes with degree π that *p*". The operators satisfy axioms N, K, D, PI and NI given in the main text. We also define an indication relation $\underset{r,i}{\Rightarrow}$ with the formulae $p\underset{r,i}{\Rightarrow}q$ reading "*p* indicates *q* to *i* on the basis of reasoning *r*". For simplicity, assume that the set *R* of available reasoning modes is a singleton. The indication relation satisfies axioms I1, I2, I3, I4 and SR given in the main text. Throughout, I assume that the players are Bayesian rational, which we can capture with the following proposition:

(BR) $D_i s \leftrightarrow s \in \max_{s\in A_i} E[u_i(s)|B_i^\pi D_j s']$, with $s' \in A_j$.

Moreover, I assume that the players commonly believe with degree 1 proposition BR.

Take the following formulae:

$p \underset{r,1}{\Rightarrow} D_2 v \qquad p \underset{r,2}{\Rightarrow} D_1 x$

$q \underset{r,1}{\Rightarrow} D_2 z \qquad q \underset{r,2}{\Rightarrow} D_1 y$

$D_2 v \underset{r,1}{\Rightarrow} B_2^{\pi'} D_1 x \qquad D_1 x \underset{r,2}{\Rightarrow} B_1^{\pi} D_2 v$

$D_2 z \underset{r,1}{\Rightarrow} B_2^{\pi'} D_1 y \qquad D_1 y \underset{r,2}{\Rightarrow} B_1^{\pi} D_2 z$

From them, using I4, we can construct another set of formulae:

$p \underset{r,1}{\Rightarrow} B_2^{\pi'} D_1 x$

$p \underset{r,2}{\Rightarrow} B_1^{\pi} D_2 v$

$q \underset{r,1}{\Rightarrow} B_2^{\pi'} D_1 y$

$q \underset{r,2}{\Rightarrow} B_1^{\pi} D_2 z$

Therefore, we have

$p \underset{r,1}{\Rightarrow} a = (D_2 v \wedge B_2^{\pi'} D_1 x)$

$p \underset{r,2}{\Rightarrow} b = (D_1 x \wedge B_1^{\pi} D_2 v)$

$q \underset{r,1}{\Rightarrow} c = (D_2 z \wedge B_2^{\pi'} D_1 y)$

$q \underset{r,2}{\Rightarrow} d = (D_1 y \wedge B_1^{\pi} D_2 z)$

Finally, define

$p \longrightarrow (B_N^1 p \wedge B_N^{f_r(1)} B_N^1 p)$ $\qquad$ $q \longrightarrow (B_N^1 q \wedge B_N^{f_r(1)} B_N^1 q)$

Note that the last two sets of formulae correspond to assumptions L1, L2 and L3 in the main text. By axioms I1, I3 and SR, we thus have (B_* is the common belief operator defined by the infinite sequence B_N, $B_N B_N \ldots$)

$p \longrightarrow B_*^{f_r(1)}(a \wedge b)$ $\qquad$ $q \longrightarrow B_*^{f_r(1)}(c \wedge d)$

This shows that if p (resp. q) is true, then proposition a and b (resp. c and d) are commonly believed with degree $f_r(1)$ in the population. Of course, as proposition $a \wedge b$ (resp. $c \wedge d$) includes the strategy profile played, the latter is also commonly believed. Moreover, common belief in BR implies that

$D_1 x \leftrightarrow x \in \max_{s \in A_1} E[u_1(x) | B_*^{f_r(1)}(a \wedge b)]$

$D_1 y \leftrightarrow y \in \max_{s \in A_1} E[u_1(y) | B_*^{f_r(1)}(c \wedge d)]$

$D_2 v \leftrightarrow v \in \max_{s \in A_2} E[u_2(v) | B_*^{f_r(1)}(a \wedge b)]$

$D_2 z \leftrightarrow z \in \max_{s \in A_2} E[u_2(z) | B_*^{f_r(1)}(c \wedge d)]$

It may be useful to summarize the syntax in the following matrices:

If *p*, then:

	$D_2\, v \wedge B_2^{\pi'} D_1 x$	$D_2 z \wedge B_2^{\pi'} D_1 y$
1	$B_1^{fr(1)}$	$B_1^{fr(0)}$

	$D_1 x \wedge B_1^{\pi} D_2 v$	$D_1 y \wedge B_1^{\pi} D_2 z$
2	$B_2^{fr(1)}$	$B_2^{fr(0)}$

If *q*, then

	$D_2\, v \wedge B_2^{\pi'} D_1 x$	$D_2 z \wedge B_2^{\pi'} D_1 y$
1	$B_1^{fr(0)}$	$B_1^{fr(1)}$

	$D_1 x \wedge B_1^{\pi} D_2 v$	$D_1 y \wedge B_1^{\pi} D_2 z$
2	$B_2^{fr(0)}$	$B_2^{fr(1)}$

These tables correspond to infinite belief hierarchies where the actual game situation (p or q) defines the players' "types". For instance, at p player 1 is of type t_1x: he plays x and believes with probability $B_1^{fr(1)}$ that player 2 of type t_2v and plays v. Moreover, he believes with probability $B_1^{fr(1)}$ that player 2 believes that he is of type t_1x and that he plays x, and so on. Each game situation thus corresponds to a "type-profile" $t = (t_1, t_2)$. As both players are symmetric reasoners with respect to each game situation, this implies that they commonly believe that they infer the same practical and epistemic conclusions, provided they believe that there is mutual belief over what is the game situation. In other words, the players commonly believe that their types are correlated with either $t(p) = (t_1x, t_2v)$ or $t(q) = (t_1y, t_2z)$. As a final point, it is worth noting that since both p and q indicate commonly believed belief hierarchies and that we have assumed that both the game and Bayesian rationality are also commonly believed, the resulting strategy profiles are necessarily (pure or mixed) Nash equilibria. Therefore, the conjectures

($B_1^{fr(1)}$; $B_2^{fr(1)}$) must themselves be in equilibrium (see [46]). Note that in this specific case we know that the Nash equilibria will be in pure strategy, *i.e.* $f_r(1) = f_r(1) = 1$. This is due to the fact that the only mixed-strategy Nash equilibria cannot hold in both p and q as $f_r(\pi) > ½$ and $f_r(\pi') > ½$ are required.

The task is now to translate this syntax into a *s.e.m.* where the CPA holds. This translation is done by defining a truth value function V mapping any proposition p at any state w onto the truth value "true" or "false". The clauses defining the semantical relation with the syntax for any model M are the following traditional ones:

$(M, w) \vDash p$ iff $V(w, p) = true$

$(M, w) \vDash \neg p$ iff $(M, w) \nvDash p$

$(M, w) \vDash p \wedge q$ iff $(M, w) \vDash p$ and $(M, w) \vDash q$

To define the semantic belief operators $\boldsymbol{B}_i^{\pi}$, it is necessary first to specify the possibility operators $\mathscr{P}_i$ and possibility sets $P_i w$. We assume that they satisfy axiom A2 of the main text, which implies that the players have an information partition I_i. However, as we are exclusively reasoning in terms of belief and not of knowledge, we do not impose axiom A1. Instead, it is only required that the players' beliefs be consistent:

(A1') $\forall w \in W: P_i w \neq \emptyset$.

We ascribe to each player a (not necessarily common) probability measure π_i over the state space W with $\pi_{i,w}$ the standard conditional probability at w. Finally, denote $|p|$ the set of states w for which $(M, w) \vDash p$. The semantic belief operators can then be defined in the following way

$(M, w) \vDash B_i^1 p$ iff for all $w' \in P_i w$, $(M, w') \vDash p$.

$(M, w) \vDash B_i^{\pi} p$ iff for at least one $w' \in P_i w$, $(M, w') \vDash p$, with $\pi = \pi_{i,w}|p|$.

$\boldsymbol{B}_i^1(E) = \{w | P_i w \in E\}$.

$\boldsymbol{B}_i^{\pi}(E) = \{w \big| P_i w \cap E \neq \emptyset, \pi = \pi_{i,w}(E)\big\}$.

The common-belief operator $\boldsymbol{B}_*^{\pi}$ is defined on a similar basis than the common knowledge operator in section 4:

$\mathscr{P}^*(w) = P^*w = \bigcup_{k=1}^{\infty} P_k^* w$,

with $P_1^* w = \bigcup_{i \in N} P_i w$ and $P_k^* w = \bigcup \{P_1^* w' | w' \in P_{k-1}^* w\}$.

$\boldsymbol{B}_*^{\pi}(E) = \{w \mid P^*w \cap E \neq \emptyset, \pi = \pi_{i,w}(E) \text{ for all } i\}$.

Recalling that $|p|$ is the event that p, consider a first *s.e.m.* $\mathscr{I}$: $< W, \{R_i, S_i, \pi_i\}_{i \in N} >$ with $W = |p|$. By assumption, we set $\boldsymbol{B}_i^1 |p|$ for both players. The following obtains:

$S_1(w) = x$ at all $w \in W$.

$S_2(w) = v$ at all $w \in W$.

$w \in \boldsymbol{B}_{\mathbf{1}}^{f_r(\boldsymbol{\pi})=\mathbf{1}} D_2 v$ at all $w \in W$.

$w \in \boldsymbol{B}_{\mathbf{2}}^{f_r(\boldsymbol{\pi}')=\mathbf{1}} D_1 x$ at all $w \in W$.

$w \in \boldsymbol{B}_{*}^{f_r(\mathbf{1})=\mathbf{1}} |p|$ at all $w \in W$.

The same applies with event $|q|$. The last proposition can be seen as a restatement of Aumann's claim that the players' information partitions and priors are "common knowledge". We can now define another *s.e.m.* with $W = |p| \cup |q|$. Note that by assumption $|p|$ and $|q|$ are mutually exclusive as both events cannot be simultaneously commonly believed with probability superior to ½. Therefore, it can be checked that we have

$\boldsymbol{B}_{\boldsymbol{i}}^{\mathbf{1}} |p \cup q|$for all i.

$w \in \boldsymbol{B}_{*}^{f_r(\mathbf{1})=\mathbf{1}} |p| = |p|$.

$w \in \boldsymbol{B}_{*}^{f_r(\mathbf{1})=\mathbf{1}} |q| = |q|$.

Denote γ any exogenously given probability measure over the probability space $\Gamma = (W, \gamma)$. Define a function $g: \Gamma \rightarrow A$ with $prob\{g^{-1}(s)\} = \gamma$, and where s is any of the four possible strategy profiles $[(x ; v), (x ; z), (y ; v), (y ; z)]$. In the case where the above belief hierarchies define pure Nash equilibria, $\boldsymbol{B}_{\boldsymbol{i}}^{\mathbf{1}} s$ for $i = 1, 2$ with $s = (x ; v)$ if p and $s = (y ; z)$ if q. Then $|p|$ and $|q|$ are singletons and it follows quite straightforwardly that there is a common prior *cp* corresponding to

For $w = |p|$, $cp(w) = \frac{prob\{g^{-1}(x\,;v)\}.\boldsymbol{B}_{*}^{fr(\mathbf{1})}|p|.\boldsymbol{B}_{\boldsymbol{i}}^{\mathbf{1}}(x\,;v)}{prob\{g^{-1}(x\,;v)\}.\boldsymbol{B}_{*}^{fr(\mathbf{1})}|p|.\boldsymbol{B}_{\boldsymbol{i}}^{\mathbf{1}}(x\,;v)+\text{prob}\{g^{-1}(y\,;z)\}.\boldsymbol{B}_{*}^{fr(\mathbf{1})}|\text{q}|.\boldsymbol{B}_{\boldsymbol{i}}^{\mathbf{1}}(y\,;z)} = \frac{prob\{g^{-1}(x\,;v)\}}{prob\{g^{-1}(x\,;v)\}+\text{prob}\{g^{-1}(y\,;z)\}}$ for $i = 1, 2$.

For $w = |q|$, $cp(w) = \frac{prob\{g^{-1}(y\,;z)\}.\boldsymbol{B}_{*}^{fr(\mathbf{1})}|p|.\boldsymbol{B}_{\boldsymbol{i}}^{\mathbf{1}}(y\,;z)}{prob\{g^{-1}(x\,;v)\}.\boldsymbol{B}_{*}^{fr(\mathbf{1})}|p|.\boldsymbol{B}_{\boldsymbol{i}}^{\mathbf{1}}(x\,;v)+\text{prob}\{g^{-1}(y\,;z)\}.\boldsymbol{B}_{*}^{fr(\mathbf{1})}|\text{q}|.\boldsymbol{B}_{\boldsymbol{i}}^{\mathbf{1}}(y\,;z)} = \frac{prob\{g^{-1}(y\,;z)\}}{prob\{g^{-1}(x\,;v)\}+\text{prob}\{g^{-1}(y\,;z)\}}$ for $i = 1, 2$.

A similar reasoning applies if the belief hierarchies define Nash equilibria in mixed-strategy, though we have ruled out this case here. In this case, $|p| = |q|$ and the common prior is computed as usual by multiplying the players' unconditional beliefs $\boldsymbol{B}_{\mathbf{1}}^{\alpha}$ and $\boldsymbol{B}_{\mathbf{2}}^{\beta}$ with $\alpha, \beta > 0$ for the events $|D_1x, D_2v|$, $|D_1x, D_2z|$, $|D_1y, D_2v|$ and $|D_1y, D_2z|$.

This result is not surprising. Indeed, as I have noted above, the belief hierarchies corresponding to the events $|p|$ and $|q|$ defined Nash equilibria. Any probability distribution of Nash equilibria in a game is necessarily a correlated equilibrium distribution and Aumann's theorem implies that for any correlated equilibrium in a game G, we can construct a *s.e.m.* where the players are Bayesian rational at all w and have a common prior over the state space. We have thus proved that symmetric reasoning entails the CPA. Note however the key difference: in the underlying syntax, I have nowhere assumed that the players hold some kind of "prior belief". The first probability measure $\pi = 1$ is simply a statement for what the players believe with certainty (either event $|p|$ or event $|q|$) and the second $f_r(\pi)$ reflects the players' belief hierarchies generated by their reasoning mode. I do not have assumed in the syntax that the players' have a probability measure over the propositions p and q. I have added an exogenously given probability measure in the corresponding semantic model for the sole purpose of generating a common prior distribution over the whole state space. But it is clear that this probability measure is purely a mathematical artefact and is inessential to understand how the players coordinate. What matters is that the players are symmetric reasoners with respect to propositions p and q. As a final point, it is worth noting that if we stay at the semantic level, the indication relation disappears as all relations of implication are captured by the inclusion operation. For instance, we obviously have $|p| \subseteq |D_1x \wedge D_2v|$. However, from the players' point of view (and also from the modeler's one), there is absolutely nothing logical in this implication. This implication is be quite the contrary due to the indication relations $p \underset{r,1}{\Rightarrow} a = (D_2v \wedge B_2^{\pi'} D_1x)$ and $p \underset{r,2}{\Rightarrow} b = (D_1x \wedge B_1^{\pi} D_2v)$ and the fact that the players are symmetric reasoners with respect to p.

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