Simple quantitative electrostatic experiments for teachers and students

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Abstract
The article presents low-cost experiments that enable to measure or at least estimate the values of charged straws, plastic rods or similar objects. First type of these measurements uses the Coulomb’s law. Several possibilities how to calculate the charge that can be used either at high school or at university level are presented. The second method uses a capacitor and a cheap multimeter. It may elucidate the principle of charge meters provided for schools e.g. by Vernier and other companies. The series of experiments and calculations presented here can be regarded as an example of “multilayered simple experiments” approach presented at GIREP 2009 [1].

Introduction
Students often have nearly no idea about the values of charges of some things around us, e.g. charged plastic rods, straws or even people themselves, for example when walking in shoes with rubber soles at plastic flooring. This lack of knowledge concerns not only high school students but also future teachers and physics teachers in schools. Even for them it is not easy to estimate the value of charge of plastic straw after being rubbed with a piece of cloth. Of course, it can be measured by a charge meter. But do teachers and students understand such measurements? How many of them could explain the principle of a charge meter? One can worry that quite often this special instrument is perceived rather as a “black box” somehow providing some results that cannot be checked in a different way.

Yet, charge is one of the most basic concepts of electrostatics and the whole area of electricity. So it is perhaps worth for teachers and students to know at least the order of magnitude estimates of charges in some concrete situations – and to be able to support and verify these estimates by simple and understandable quantitative measurements.

In the following text two types of such experiments are presented. The first one uses the Coulomb’s law (and also, at a more sophisticated level, the Gauss’s law) to calculate the value of the charge of a plastic straw, a rod or a similar object. The simplest method of calculation that can be used at high school level provides just very rough estimate. The calculation based on the Gauss’s law gives, as we shall see, more precise results.

The second type of experiments enables to measure charge values by the same principle as is used in charge meters: i.e. by charging a capacitor of a known capacity. We shall see that a low-cost multimeter can be used in such measurements.

The experiments presented here were already used both in pre-service teacher training (in the seminar “Electricity and magnetism step by step” for future physics teachers at Charles University in Prague) and in in-service teacher training of Czech physics teachers (at the workshop at the conference “Heureka Workshops 2009” [2]).

Simple hands-on experiments
The fact that a plastic straw rubbed e.g. by a paper napkin attracts small pieces of paper or sticks itself to a wall is well known. (Sometimes people are surprised how long a straw can
stay stuck to a wall; it may be for days.) Two plastic straws charged by rubbing repel each other. In fact, we can even feel the repulsion by our own hands. Try to hold the charged straws by your fingers as it is shown at Fig. 1. Your fingers will clearly feel the force preventing to bring straws closer to each other. (Of course, there is a “trick” in it – or rather a simple application of mechanics. The straws act as levers, with long lever arms at which the repulsion acts and a short lever arms at which we hold them in our fingers. That’s why the force we perceive is greater than we would expect.)

Our first experiment was just a qualitative one. But it is easy to convert it to a version which could enable us to estimate the values of charges of the straws.

Hold the straw horizontally as it is shown at Fig. 2. Hold the lower straw tightly so its position is fixed and let the upper straw be just freely supported by your fingers at the end. (You should prevent it from slipping to the side but let it be free to move up or down.) You will see that the upper straw will “float” (or hover) above the lower one. Of course, it is due to electrostatic repulsion. And it is this simple experiment that we can use for determining the approximate values of charges of the straws.

The distance $d$ between the straws is typically about 2 cm. Sometimes, under good conditions, it may be up to 4-6 cm. (The straws should be clean and it is better to use a new paper napkin for rubbing them. It is not necessary to rub a straw many times; but the napkin should be pressed hard to a straw while rubbing.)

**How to determine $Q$**

Grasping the principle of determining the value of charge is simple even at high school level. The electrostatic repulsion force balances the weight of the upper straw. By weighting the straw or, rather, several straws, we can find that the mass of the straw is about 0,5 g. (You
should weight your own straws, the mass of a particular type may be a bit different.) So its weight – and also the value of the electrostatic force $F$ – is about 5 mN.

At high school level the Coulomb’s law

$$F = k \cdot \frac{Q_1 Q_2}{r^2} = \frac{1}{4\pi \varepsilon} \cdot \frac{Q_1 Q_2}{r^2} \quad (1)$$

is a natural starting point for determining the value of the charge $Q$. Here $Q_1$ and $Q_2$ are the values of charges, $r$ is their distance and $\varepsilon$ is a permittivity of the surrounding air that we can take equal to the permittivity of vacuum $\varepsilon_0$ – or simply take $k = 9 \cdot 10^9$ Nm$^2$/C$^2$.

If we take the charges of both straws to be approximately the same, $Q_1 = Q_2 = Q$, it follows from (1) that

$$Q = r \sqrt{\frac{F}{k}} \quad (2)$$

For the distance $r = 2 \text{ cm} = 2 \cdot 10^{-2} \text{ m}$ and $F = 5 \cdot 10^{-3} \text{ N}$ the formula (2) gives $Q = 1,5 \cdot 10^{-8} \text{ C} = 15 \text{ nC}$.

Well, it is a very simple derivation – but even a high school student should now start to protest vehemently. The Coulomb’s law is valid for point charges (or homogenously charged spheres) – and the straws are far from being point-like!

So we should take our derivation as just a very crude approximation. How could it be improved?

At university level students learn the Gauss’s law and can use it to calculate the electric field near an infinitely long uniformly charged straight line. The resulting formula is

$$E_R = \frac{\tau}{2\pi \varepsilon R} = k \frac{2\tau}{R} \quad , \quad (3)$$

where $\tau$ is the linear charge density and $R$ is the distance from the line. The force acting at a charge $Q$ at this distance is $F = Q E_R$. If we take the linear charge density at a straw of a length $L$ to be $\tau = Q / L$, we can express the charge as

$$Q = \sqrt{L R \frac{F}{2k}} \quad . \quad (4)$$

The length $L$ of the straw is about 16 cm. For the distance $R$ equal to 2 cm formula (4) gives 30 nC.

Again, the derivation of (4) (now at introductory university level) was very simple. And again, (4) is surely just an approximation – the straws are not infinitely long.

We have now two approximations. A natural question can motivate both us and the students to further calculations. Can we estimate how good our simple estimates are?

**How precise are our estimates?**

It may be a bit surprising that it is possible to derive an *exact* formula for the force between two finite homogenously charged parallel rods – and the derivation is well in the scope of introductory university level. It may be a classical end of chapter task and perhaps it is present in some textbooks. But being just stated in a textbook the task could have been perceived as boring and artificial. Now it is interesting and attractive because we are motivated! Just how
much we were in error when using such absurd approximation as taking 16 cm long straw as a point charge or an infinite line?

We will not present the detailed derivation here, just the resulting formula. It reads

\[ F = 2k \frac{Q_1 Q_2}{L^2} \left( \sqrt{1 + \frac{L^2}{d^2}} - 1 \right), \]

(5)

Where \( d \) is the distance between the straws (as it was denoted at Fig. 2). Considering again that the values of charge at both straws are the same we can derive the final expression for \( Q \):

\[ Q = d \frac{F}{2k} \left( \sqrt{1 + \frac{L^2}{d^2}} \right). \]

(6)

It can be easily seen that the approximations (2) and (4) (where the distance \( r = R = d \)) follow from (6) for \( L \ll d \) and \( d \ll L \) respectively. But it is more useful to compare results provided by different methods and approximations for the realistic values, i.e. for the straw length \( L = 16 \text{ cm} \) and the distance \( d \) from 1 to 5 cm. The comparison is presented in Tab. 1 and in Fig. 3.

| distance \( d \) of the straws | \( Q \) determined by the formula (approximation): |
|------------------------------|---------------------------------|---------------------------------|---------------------------------|
|                              | taking straws as point charges  | taking straws as infinite lines | exact result for finite rods    |
|                              | - formula (2)                   | - formula (4)                   | - formula (6)                   |
| 1 cm                         | 7.5 nC                          | 21 nC                           | 22 nC                           |
| 2 cm                         | 15 nC                           | 30 nC                           | 32 nC                           |
| 3 cm                         | 22 nC                           | 37 nC                           | 40 nC                           |
| 4 cm                         | 30 nC                           | 42 nC                           | 48 nC                           |
| 5 cm                         | 36 nC                           | 47 nC                           | 55 nC                           |

Table 1 – Comparison of charge values provided by different approximations

![Fig. 3. Relative value of charge with respect to the exact value provided by the approximation taking straws as point charges (blue points) and as infinite lines (red points).](image)

We can see that the calculation based on the Gauss’s law provides rather good approximation of the real value of charge despite the fact that the straws are far from being infinite. The error
is less than 15% for distances $d$ up to 5 cm; the approximation slightly underestimates the true value of the charge.

It may be surprising that even a very crude approximation taking straws as point charges, provides results that are not orders of magnitude wrong. This method underestimates the values of charges about twice or three times for distances in the range 1-3 cm and it is even better for larger distances. (We can discuss with students that this is partly due to the presence of a square root in the formulas.)

Even the calculation based on the Coulomb’s law could be made much more precise if we took the straw as a series of point charges. This approach will be described elsewhere.

Finally, it is worth to note (and to discuss with students) that even the “exact formula” (6) could not provide absolutely precise values of charges in real situations. (6) was derived for the case of two parallel uniformly charged infinitely thin rods of the same length. In reality the charge will not be exactly uniformly distributed, the charges of both rods may differ; the straws are not strictly parallel, the presence of our hands distorts the electric field etc. So we should be aware that our method provides just more or less precise estimates of charges on straws.

**Another method: how to measure charge by a capacitor and a cheap multimeter**

The following method can be used to illustrate the principle of charge meters. The basic idea is simple, see Fig. 4. The charge, for example from a charged can, charges a capacitor of a known capacity $C$. The voltage on the capacitor is then measured by a voltmeter.

![Fig. 4. The principle of measurement of a charge by a capacitor and a voltmeter](image)

A simple formula from high school physics, $Q = CU$, is sufficient to determine the charge $Q$. For a capacitor of the capacity $C = 1 \mu F = 10^{-6} F$ the voltage $U = 1 mV = 10^{-3} V$ corresponds to the charge $Q = 10^{-9} C = 1 nC$. Even a cheap multimeter has a sensitivity and resolution of 0.1 mV so charges in the range of several nanocoulombs to microcoulombs can be safely measured. (Of course, the real multimeter will discharge the capacitor – this problem will be discussed later.) There is also one obvious practical advantage of the above mentioned setup: if we switch the range of the multimeter to milivolts the display will show the values of charges directly in nanocoulombs.

**The real experiment and its limitations**

The above mentioned idea is very simple. So why the measurement of charges is not implemented to most of ordinary multimeters and a special charge meters must be used?

The problem was already mentioned above. The multimeter has a finite input resistance and discharges a capacitor. The input resistance of cheap multimeters (when measuring a voltage) is typically equal to 10 M$\Omega$. (Some very cheap multimeters have the input resistance just
1 MΩ, those are not suitable for our measurement at all.) The time constant of discharging the capacitor is \( \tau = RC = 10\, \text{s} \), so in 10 s the voltage falls e-times. This is quite fast but our setup can still be used for approximate measurements. In the first second the voltage drops by 10%, in two seconds by about 20%. If we read the number at the display fast enough we obtain usable results.

The real setup of our simple experiment is shown at Fig. 5. It can be seen that also this method confirms that a charge at a plastic straw is several tens of nanocoulombs. The multimeter also shows that the charge is negative.

We will not discuss here any technical details of an experimental setup. (Let us just note that the capacitor should not be electrolytic.)

Apart from an obvious disadvantages of this measurement (the necessity to read the value very quickly, the expected error of 10-20%) there are also some advantages: the simplicity of the overall setup, the clear illustration of the principle of charge meters and also the possibility to measure values of charges exceeding the range of charge sensors supplied for example with dataloggers and computer measurement systems for schools. Our setup can be therefore used for measuring charges of large plastic rods or charges of people acquired by walking at some types of floors.

**Conclusions**

The experiments and measurements described in this article can be used for teaching and learning of electricity (especially electrostatics) at various school levels, from high school to university. They are examples of “multilayered simple experiments”(see [1]) that can be used at various levels and discussed to various depths. Such experiments provide “increasing cognitive demands” that helps develop students’ knowledge and understanding.

Hopefully some of the experiments mentioned here might be used in the future in programs like “Phyware”. Up to now the earlier versions of these experiments were used in in-service teacher training of Czech physics teachers in the Heureka project [2] and also in pre-service training of future physics teachers, in both cases with a positive response. It is planned that they will be used also in other future courses for physics teachers.
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References
